

## INFORMATION TO USERS

This material was produced from a microfilm copy of the original document. While the most advanced technological means to photograph and reproduce this document have been used, the quality is heavily dependent upon the quality of the original submitted.

The following explanation of techniques is provided to help you understand markings or patterns which may appear on this reproduction.

1. The sign or "target" for pages apparently lacking from the document photographed is "Missing Page(s)". If it was possible to obtain the missing page(s) or section, they are spliced into the film along with adjacent pages. This may have necessitated cutting thru an image and duplicating adjacent pages to insure you complete continuity.
2. When an image on the film is obliterated with a large round black mark, it is an indication that the photographer suspected that the copy may have moved during exposure and thus cause a blurred image. You will find a good image of the page in the adjacent frame.
3. When a map, drawing or chart, etc., was part of the material being photographed the photographer followed a definite method in "sectioning" the material. It is customary to begin photoing at the upper left hand corner of a large sheet and to continue photoing from left to right in equal sections with a small overlap. If necessary, sectioning is continued again -- beginning below the first row and continuing on until complete.
4. The majority of users indicate that the textual content is of greatest value, however, a somewhat higher quality reproduction could be made from "photographs" if essential to the understanding of the dissertation. Silver prints of "photographs" may be ordered at additional charge by writing the Order Department, giving the catalog number, title, author and specific pages you wish reproduced.
5. PLEASE NOTE: Some pages may have indistinct print. Filmed as received.

**Xerox University Microfilms**

300 North Zeeb Road  
Ann Arbor, Michigan 48106

74-24,670

TAJCHMAN, Edwin Joseph, 1934-  
COMPUTER AIDED ANIMATION UTILIZING  
A MAPPING PROCESS.

University of Denver, Ph.D., 1974  
Engineering, electrical

University Microfilms, A XEROX Company, Ann Arbor, Michigan

© 1974

EDWIN JOSEPH TAJCHMAN

ALL RIGHTS RESERVED

COMPUTER AIDED ANIMATION UTILIZING  
A MAPPING PROCESS

---

A Dissertation  
Presented to  
The Faculty of the College of Engineering  
University of Denver

---

In Partial Fulfillment  
of the Requirement for the Degree  
Doctor of Philosophy

---

by  
Edwin J. Tajchman  
April 1974

THE UNIVERSITY OF DENVER

COLLEGE OF ENGINEERING

Upon the recommendation of the professor in charge of the thesis and of the chairman of the DEPARTMENT OF ELECTRICAL ENGINEERING, this thesis is hereby accepted in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

Maynard L. Moe  
Professor in charge of thesis

John A. Weese  
Dean of Engineering

May 29, 1974  
Date

## TABLE OF CONTENTS

| CHAPTER |                                               | PAGE |
|---------|-----------------------------------------------|------|
| 1.      | INTRODUCTION. . . . .                         | 1    |
| 2.      | DEVELOPMENT OF ANIMATION. . . . .             | 4    |
|         | 2.1 Historical Background. . . . .            | 4    |
|         | 2.2 Simplifying Animation Methods. . . . .    | 6    |
|         | 2.3 Uses Other Than Entertainment. . . . .    | 9    |
| 3.      | TECHNICAL AIDS FOR ANIMATION. . . . .         | 12   |
|         | 3.1 Aids for Hand Drawn Animation. . . . .    | 12   |
|         | 3.2 Use of Computers in Animation. . . . .    | 17   |
| 4.      | CONTROLLING MOVEMENT BY MAPPING . . . . .     | 26   |
|         | 4.1 Concept. . . . .                          | 26   |
|         | 4.2 Developing the Basic Mapping              |      |
|         | Operators . . . . .                           | 29   |
|         | 4.3 Shaping Images with the Mapping           |      |
|         | Operators . . . . .                           | 36   |
|         | 4.4 Moving Images with Time Varying           |      |
|         | Operators . . . . .                           | 43   |
| 5.      | ANIMATION BY USE OF MULTIPLE MAPPING          |      |
|         | OPERATORS. . . . .                            | 51   |
|         | 5.1 Achieving Character Movement . . . . .    | 51   |
|         | 5.2 Fairing . . . . .                         | 56   |
| 6.      | IMPLEMENTING THE ANIMATION EQUATION . . . . . | 60   |
|         | 6.1 A Raster as an Electronic Plane. . . . .  | 60   |
|         | 6.2 Producing the Plane . . . . .             | 62   |
|         | 6.3 Magnification Operator. . . . .           | 63   |

| CHAPTER |                                                                 | PAGE |
|---------|-----------------------------------------------------------------|------|
|         | 6.4 Translation Operator. . . . .                               | 63   |
|         | 6.5 Rotation Operator . . . . .                                 | 65   |
|         | 6.6 Positioning Translation Operator. . . . .                   | 66   |
|         | 6.7 Determining Operator Arguments. . . . .                     | 68   |
| 7.      | ANIMATION BY MAPPING . . . . .                                  | 72   |
|         | 7.1 Requirement of Animation. . . . .                           | 72   |
|         | 7.2 Compound Programming. . . . .                               | 75   |
|         | 7.3 Methods of Implementing Compound<br>Operators. . . . .      | 85   |
|         | 7.4 Hierarchical Programming. . . . .                           | 87   |
|         | 7.5 Implementation of an Hierarchical<br>Walk Program . . . . . | 95   |
| 8.      | SUMMARY AND CONCLUSIONS. . . . .                                | 106  |
|         | 8.1 Summary . . . . .                                           | 106  |
|         | 8.2 Areas for Research. . . . .                                 | 107  |
|         | 8.3 Conclusions . . . . .                                       | 110  |
| 9.      | APPENDIX A CAESAR . . . . .                                     | 112  |
| 10.     | BIBLIOGRAPHY . . . . .                                          | 119  |

## LIST OF ILLUSTRATIONS

| FIGURE |                                                                                                                                                     | PAGE |
|--------|-----------------------------------------------------------------------------------------------------------------------------------------------------|------|
| 4.1    | Shaping Effect on Rectangular Planes of<br>Vertical Shaping Translation Functions<br>Which are Sine Waves of Different<br>Frequencies . . . . .     | 38   |
| 4.2    | Shaping Effect on Rectangular Planes of<br>Horizontal Shaping Translation Functions<br>Which are Cosine Waves of Different<br>Frequencies . . . . . | 39   |
| 4.3    | Bending an Arm by Employing a Triangle<br>Waveform in a Translation Operator. . . . .                                                               | 41   |
| 4.4    | Effect of Horizontal Size on Maintaining<br>Proper Perspective During Bending . . . . .                                                             | 46   |
| 4.5    | Three Key Frames for a "Knocking" Sequence. . .                                                                                                     | 49   |
| 4.6    | Seven Key Poses for "Knocking" Animation. . . .                                                                                                     | 50   |
| 5.1    | Input Image for Character . . . . .                                                                                                                 | 52   |
| 5.2    | Output Image of Assembled Character . . . . .                                                                                                       | 53   |
| 5.3    | One Type of Fairing Function. . . . .                                                                                                               | 59   |
| 6.1    | Block Diagram of a Raster Generator . . . . .                                                                                                       | 64   |
| 6.2    | Block Diagram of a Magnifying Operator. . . . .                                                                                                     | 64   |
| 6.3    | Block Diagram of a Translation Operator . . . .                                                                                                     | 67   |
| 6.4    | Block Diagram of a Rotation Operator. . . . .                                                                                                       | 67   |
| 7.1    | Artwork for Arm Oriented Horizontally on<br>Section . . . . .                                                                                       | 78   |
| 7.2    | Arm Bent Using Only Vertical<br>Bending Operator. . . . .                                                                                           | 78   |

| FIGURE |                                                                               | PAGE |
|--------|-------------------------------------------------------------------------------|------|
| 7.3    | Arm Distorted by Raster Shaping. . . . .                                      | 79   |
| 7.4    | Arm Bent by Raster Shaping and Vertical<br>Bending Operator . . . . .         | 79   |
| 7.5    | Arm Bent With a Fixed Amplitude Bending<br>Translation. . . . .               | 80   |
| 7.6    | Arm Bent With a Dependent Amplitude<br>Bending Translation. . . . .           | 80   |
| 7.7    | Artwork of Arm With No Bend. . . . .                                          | 82   |
| 7.8    | 90 Degree Bend With Vertical Translation<br>Operator Only. . . . .            | 82   |
| 7.9    | 135 Degree Bend With Vertical Translation<br>Operator Only. . . . .           | 83   |
| 7.10   | 30 Degree Bend With Compound Bend Operator . . .                              | 83   |
| 7.11   | 90 Degree Bend With Compound Bend Operator . . .                              | 84   |
| 7.12   | 150 Degree Bend With Compound Bend Operator. . .                              | 84   |
| 7.13   | Effect of Rotation About Horizontal Axis by<br>Varying Vertical Size. . . . . | 88   |
| 7.14   | Effect of Rotation With Compound Operator. . . .                              | 88   |
| 7.15   | Eight Key Frames of a Typical Cartoon<br>Character Walk Cycle . . . . .       | 91   |
| 7.16   | Input Artwork Configuration for Pierre . . . . .                              | 101  |
| 7.17   | Pierre as Assembled. . . . .                                                  | 101  |
| 7.18   | First Key Frame of a Walk Cycle. . . . .                                      | 102  |
| 7.19   | Second Key Frame of a Walk Cycle . . . . .                                    | 102  |
| 7.20   | Third Key Frame of a Walk Cycle. . . . .                                      | 103  |
| 7.21   | Fourth Key Frame of a Walk Cycle . . . . .                                    | 103  |



| FIGURE |                                                | PAGE |
|--------|------------------------------------------------|------|
| 7.22   | Fifth Key Frame of a Walk Cycle. . . . .       | 104  |
| 7.23   | Sixth Key Frame of a Walk Cycle. . . . .       | 104  |
| 7.24   | Seventh Key Frame of a Walk Cycle. . . . .     | 105  |
| 7.25   | Eighth Key Frame of a Walk Cycle . . . . .     | 105  |
| 9.1    | CAESAR animation System Block Diagram. . . . . | 117  |
| 9.2    | Control Console for CAESAR . . . . .           | 118  |

## ACKNOWLEDGEMENTS

I wish to express my appreciation to Computer Image Corporation for support of the work which led to this dissertation.

My contribution to this effort was in the development of the concept of mapping to describe the animation process and the use of a hybrid computer to realize this process.

A major company effort was required to convert the concept into a working tool for animators. I wish to thank Lee Harrison III, President of Computer Image, for his continuous support of my effort, Francis Honey for his help in discovering methods of implementing the concept, Marshall Parker who designed the digital portion of CAESAR, Dave Wells who designed the special digitally controlled animation oscillators, Bill Altemus who designed the overlap system and with Jim Duca developed the colorizing technique, and Domenic Iaia who provided the artistic talent, all for assistance in proving the feasibility of CAESAR. I also wish to thank Dr. Maynard Moe for his patient counsel throughout the writing of this dissertation.

Film or videotape demonstrations of the type of animation produced by the CAESAR system can be arranged by contacting:

Computer Image Corporation  
2475 West Second Avenue  
Denver, Colorado 80223

## 1. INTRODUCTION

Animation is one of those craft products that has fallen from the levels of excellence once attained. From its humble beginning it matured into a craft that was a new method of communication. Visions that once had existed only in an artist's mind could now be brought to life.

Many creative people contributed to the development of animation. However, perhaps the peak of technical capability was achieved by the Disney studios in the 1930s, 1940s and the 1950s. Classic animated films such as "Snow White and the Seven Dwarfs", "Bambi" and "Fantasia" were produced by 24 different pieces of art for every second of animated imagery. The quality of the character movement and the detail in the imagery were due to the amount of care and effort put out by Disney's skilled animators. This quality had a very high cost in terms of man hours required to produce the necessary drawings.

Animation soon had proven its value in entertainment, education, training and advertising. However, its use had not greatly increased in the last two decades. The cost of animation produced by conventional techniques limited its expanded use. Only the advertising business could still afford high quality, conventionally produced animation.

Since creativity still exists, many innovative people have tried to apply technology to solve the problem of high priced animation. Many labor saving techniques have been developed to aid conventionally produced animation. However, a solution that did not require the artist to create all of the required art by hand became the goal. In these days of automation and computers it seemed quite

possible that some method could be devised to eliminate much of the tedium of animation. Then the artist could concentrate on creating the total effect and not be distracted by the need to draw many similar pictures.

The first efforts of using computers to help produce animation led to the development of special languages. These made it easier for those interested to produce animation which consisted of abstract patterns or crude line drawings of characters. Some used the languages to produce striking examples of simulation, obtaining in a visual form the output of a computer performing scientific calculations.

As interesting as all of these efforts were, there were very few animators or artists using these new tools. They were forced to learn new disciplines in order to use them and then had to accept results that were quite different from those to which they had become accustomed. The character of Snow White was not produced by a line drawing nor was it easy to write an equation resulting in Donald Duck's face. Therefore, while the advances make it possible to produce excellent animated films for certain technical subjects, the general animation problem is still lacking a solution.

The specific problem addressed in this investigation is the development of a technique that is useful for the creation of animation by employing a computer which utilizes a mapping operation. This technique must meet certain requirements if it is to be effective and thereby be useful to artists producing animation.

The first requirement is that an artist should be able to create his characters by using his acquired skills and experience,

that is, by drawing them. Another requirement is that the artist should then be able to control the character's movements with simple directions. Also, the animation being created should be produced in a manner that can give visual feedback to the artist. Next, it should be easy to change or modify the animation being created so that the timing and type of movement could be refined to accurately portray the desired action. Finally, the technique should make it possible for someone who is not an experienced animator to also make his thoughts visible. The sophistication of tightly timed action might not be possible by an inexperienced animator but basic movement sufficient to transmit an idea should be possible.

The effectiveness of this mapping technique will be determined by considering the kind and variety of movement possible by mapping artwork from a static plane onto a series of planes shaped by the animator.

There is a system used in the animation industry of producing animation by first drawing a series of "key" frames and then filling in between them. The kind of "key" frames that can be produced by a computer utilizing this mapping operation from a single artist's drawing will be shown for comparison to conventional "key" frames.

## 2. DEVELOPMENT OF ANIMATION

### 2.1 Historical Background

Animation is an old concept dating back to prehistoric times. Early artists gave life to painting of creatures by painting multiple sets of legs. These served to give the feeling that the creature was running rather than merely pictured at one particular instant of time. Even then, over 300 centuries ago, artists felt the need to make their pictures represent movement.

Animation's next development was due to the discovery of the natural phenomenon of persistence of vision. This is the characteristic of the human eye that makes the brain "see" an image for a brief period of time after the image is actually removed. Ptolemy, the Egyptian scientist, first noted this effect in A.D. 130 while watching sentries swing their lanterns in arcs. One lantern could appear to be one circle of light if it was moved rapidly enough (Madsen, 1969). This characteristic makes it possible to give life to a series of static drawings. If each successive drawing changes only slightly from the preceding one and if each drawing is viewed in rapid succession then the brain interprets the entire sequence as one produced by actual movement.

A number of artist-scientists developed various devices to use persistence of vision to create the illusion of movement. Usually they were some kind of circular device which is ideally suited to create repetitive movements. Examples are the Phenakistoscope which used two disks, one with pictures and the other with slots and the Daedaleum which consisted of a wide shallow cylinder with slits and drawings (Falk, 1941). The animated figure could

appear to run, jump, fly or to perform some other cyclical action.

Thomas Edison invented the movie camera around the turn of the century. Edison's movie camera coupled with a practical method of displaying the images thereby captured on film made possible the use of animation for something other than just a toy. Now it was possible to photograph a series of drawings and then to successively display the images to produce the illusion of movement.

Emile Cohl of France was one of the first recorded artists to use animation as a method for entertainment. His white stick figures jumped around on a black background to usher in an era that ran from 1908 to 1917. During this time audiences accepted animation as entertainment and demanded more. This was the age of "Gertie the Trained Dinosaur", "Felix the Cat" and "Koko the Clown."

Walt Disney led the world into the next era when he produced the first talking animated film, "Steamboat Willie." His studio took part in and led the great utilization of animation for theatre entertainment in the years 1928 to 1938. By then he had produced the first full length commercial animated cartoon, "Snow White and the Seven Dwarfs." He went on to make many more animated full length features but perhaps his greatest achievement was the production of "Fantasia." This film was a real classic with all types of creatures dancing around to music by Beethoven, Tchaikovsky, Stravinsky and Ponchielli. "Fantasia" was animated in the form known as full animation. This type requires 24 images, each slightly different from any other, to create one second of the movie.

## 2.2 Simplifying Animation Methods

Now that the world was willing to accept animation as a means of communication or entertainment many artists felt free to experiment. Where Disney's characters moved very realistically, being closely related to the movements of real models, now some characters were flat, with their movements almost abstractions. UPA (United Productions of America) was one of the first organizations to capitalize on this technique with their "Gerald McBoing Boing." (Madsen, 1969) This type of animation was simpler to create and therefore was greatly exploited in the post war era as rising labor costs pushed full animation beyond reach of the entertainment field.

As the cost of full animation began to price animated cartoons out of the theatrical market a new factor entered. Television became popular in the late forties and the fifties. Its voracious appetite consumed material at a great rate. A natural source of material was theatrical productions; movies and, for the children, animated cartoons. Many cartoons that had been produced for use in movie theaters were used to entertain children on television and to sell the advertiser's product. The price of these cartoons was too high for more of them to be produced directly for the television market. Few advertisers were willing or able to pay the price for full animation.

This new factor, that of cost, gave added impetus to a new type of animation. A number of production houses became very skilled in producing animated cartoons that were satisfactory for the television market and that could be produced at a low enough



cost to be attractive to advertisers. One reason for the acceptability of the limited animation cartoons for television was the limitation of the medium itself. The television screen is relatively small and has low resolution when compared to a movie theater screen. It is useless to attempt to achieve the same subtle effects on the television screen as are possible on a movie screen. The images can be simple and still be effective as much less is expected of television than of the movie screen.

Several different changes in animation techniques share the credit for the achievement of lower cost animation. One was the discovery that 24 different drawings for each second of action were not necessary. Double framing, exposing each drawing on two successive frames, was often very adequate to convey the impression of action. In this case only 12 different drawings were necessary for each second of animation, a 50 percent reduction in labor. Some animators cut the drawing requirements even more and filmed on "threes" where only 8 drawings per second are required. However, filming on "threes" is a harsh test of an animator's skill and will point out errors in animation quicker than filming on "twos" or "ones" so that it is not often used.

Another animation technique that provides the illusion of movement with few drawings is that of moving only certain portions of a figure or a scene. In a scene containing multiple characters only one will move at one instant of time. Then perhaps only the facial features will move if the character is speaking or additionally an arm. If the character is walking, only the legs may move with the rest of the torso only translating in order to maintain

integrity with the legs.

This second technique was enabled by the invention of the process of cell animation by Earl Hurd which he patented in June, 1915. (Halas, 1959) Different portions of the animated figure or figures are drawn on different cells, sheets of transparent celluloid. As an example, take the scene of a man walking in front of a house. Before cell animation each second of animation would require 24 or 12 separate drawings of the house with the man in a slightly different pose in each drawing. Since the development of cell animation only one drawing of the house is necessary. Only the figure need be redrawn in its different poses for each different frame of animation. The background can be photographed through the clear celluloid of each of the figure cells as they are successively placed over it. Latest techniques in animation have produced further cuts in the amount of drawing required. For the walking man, one cell would have his upper torso drawn on it without arms. The various poses of the walking man would then be composed of the cell of the torso plus a cell containing the correct pose of the arms and legs. This means that only the arms and legs need to be redrawn for each individual pose, a tremendous savings of an animator's time which translates into money.

Walter Lantz, one of the pioneers of animated films for entertainment has developed many characters. One of his most famous is Woody Woodpecker who is still in production after 30 years. Mr. Lantz says that there is a factor of five difference in the amount of film the animators in his studio can turn out for theatrical release or for use on television. (Franken, 1971) This fact

indicates the wide range of animation that is being produced. However, the high cost of animation for theatrical release, around \$10,000 per minute, has greatly reduced its market and very few short subjects like "Woody Woodpecker," "Bugs Bunny" and "Mr. Magoo" are still in production.

The Walt Disney studios have continued to produce animated feature films of excellent quality. Also, just recently a different type of animated film has arrived on the scene. This is "Fritz, the Cat", the first adult animated film. These films have proven to be popular enough so that some of them will continue to be produced. However, most of the animated films produced for the theatrical market will probably be features instead of short subjects.

### 2.3 Uses Other Than Entertainment

Another market area that has always appeared very promising and still does is the education market. Many relatively new teaching aids, movies or videotapes, computers for programmed teaching, closed circuit television, etc. are expected to help improve the quality and quantity of education. Movies or video tapes can be either live action or animated or both.

"Much of our world, physically and psychologically, is increasingly abstract, nonverbal and nonphotographable. The animation camera is a remarkable facility for showing not only what things are but also what they mean." (Madsen, 1969) This ability to describe events or processes that can not actually be recorded is a unique and valuable tool. Visualizations of chemical reactions, the theory of electron flow or the workings of the organs

of the human body are all possible by use of animation. An animated film describing the theory of an oscilloscope or of an internal combustion engine will teach these theories with better efficiency than just pictures and words in a book. The Armed Forces of the U.S. are steady users of animated films for instructional purposes. (Madsen, 1973) They have developed some very good training programs by using the best teaching techniques available including animation.

Animated film has not been used very extensively by the public educational system. The main reason for this is that the cost of animation is too high for school budgets. Simple economics decree that animation produced by traditional methods will be too expensive for potential users. For animation to be used extensively some techniques that will cut costs must be developed.

Industrial training is another area that uses animation and would use much more if it were readily available. Various processes from forming metal parts by machinery to preparing a payroll with a digital computer can be taught with the use of animation. Concepts in the fields of banking, real estate and securities can be presented with animation.

Advertising on television has been a major factor in the field of animation. The economics of advertising have made it rank much higher in importance than the amount of material it requires would seem to indicate. For example, animation for television programming carries a price tag of about \$2,000 per minute. Higher quality animation for theatrical release has a price tag of about \$10,000 per minute. However, animation for television advertising easily commands \$20,000 per minute.

An advantage of animation for advertising is in its use of invented characters. Live actors recommending a specific product frequently present an insincere image. The artificiality surrounding many of the situations is often recognized by the public. An animated commercial does not propose false relationships between the character promoting the product and the audience. (Halas, 1967)

Animation can quickly move into the world of imagination using visual humor or graphic art to turn the sales pitch into genuine entertainment. The larger budgets for a commercial allow the animator to try new techniques or to perfect ideas. Therefore, an animated commercial can often lead the state of the art. In fact, most of the time it must. Commercials depend on their uniqueness, to catch the viewer's eye. While virtually hours of material can be and are produced by utilizing a specific technique for a television cartoon series, each 60 second commercial requires a variation of a technique. Nobody wants a commercial just like one that already exists. Therefore new visuals are always in demand for animated commercials.

### 3. TECHNOLOGICAL AIDS FOR ANIMATION

#### 3.1 Aids for Hand Drawn Animation

The use of animation in all fields mentioned is limited by one main factor, cost. It was developed as a technique that required much hard work. The numerous drawings needed to show movement had to be produced by people. In times past when salaries were low this was an acceptable cost. However as living standards increased meaning higher wages for artists, the cost of animation increased. A balance was achieved between cost and use of animation at a point that has kept the use of animation from growing as much as users would like.

This gave added impetus to a search for automating animation. Since the cost of the human labor required was such a large factor, many people tried to reduce it by various means. Many different mechanical aids have been tested and some have achieved a degree of success.

As an example, consider the Animascope process. This process utilizes live actors as the principle animators.

The actors are transformed by the use of make up and costumes into the characters to be animated. They are then filmed while performing the desired action. The actor may run, dance, fall, talk, sing or perform any type of action.

Then the Animascope process is used to convert the film of the actors into line drawings on clear cells. The lines conform to the outline of the actors and also separate areas of different colors or textures. Now the cells can be painted to achieve the desired type of character. Conventional animation procedures can

then be used to combine any arbitrary background with the character animation. This involves the use of an animation stand and an animation camera.

Advantages of the system are the creation of all of the necessary line drawings of a character without the use of the laborous hand drawing process. Also, extremely accurate representations of complex movement are possible since real action produces the basic figure outlines.

One disadvantage is that the animated figures can only move within limits dictated by human movements. The desired exaggerated motions of make believe characters are difficult to achieve. Also, while this technique works well with life-like characters or objects it will not produce drawings of horses walking on their hind legs or talking. Only action that can happen in the real world can be captured on the cells. However, it can be a very effective method to produce a certain type of animation.

Another example is the De Joux process. Jean De Joux invented this system which was designed to produce animation faster and thereby cheaper. "The De Joux system allows the animator to draw only six to eight drawings and bring into play the action and movements normally possible only with 24 or more. The in-betweens are all produced optically. The system involves a special drawing table with a built-in playback unit, an editing and a color table, the De Joux optical console and multi-level projector, a computer specifically programmed for animation processing and/or video cameras and a video tape recorder.

"The table incorporates two reels of Eastman Kodak 70mm

film with a frosted matte surface on one side to accept a wider variety of drawing materials mounted side by side, which are driven horizontally across the table by electronic mechanisms controlled by the animator. The image of a completed frame is projected onto the ground glass of the next frame aperture, which enables the animator to advance his action with a subsequent drawing directly on the film.

"The system features an instant playback unit, a modular element built into the table. The unit affords the artist with the fingertip control allowing him to preview, change, recreate, modify, and expand his work moments after it's drawn.

"Optical Systems has also developed an editing and color table which enables the artist to apply color to the animator's work in a fraction of the normal time. Colors are applied with transparent inks which can be blotted dry instantly.

"The De Joux optical console utilizes rotating polarizing filters and a semisilvered beam splitter to produce the dissolves which create the illusion of motion. It performs a computer controlled playback function which, when synchronized with a conventional animation camera or video camera, enables the conversion of the output of the unit into video tape or any size motion picture film 8mm, super 8mm, 16mm or 35mm." (De Joux, 1971)

This system does decrease the number of pictures that must be drawn by the animator. However, the drawing surface that the animator must work on is 70mm film which is much smaller than most artists prefer. The accuracy and possible detail are limited by the size of the drawing surface.



Aniforms, Inc. has developed an animation process employing characters made of a special foam-like material. These characters are manipulated through wires by one or more operators, and the action is recorded by conventional film or TV cameras. This technique is very similar to producing animation by filming puppets. This is a valid process but is quite limited in the type of movements possible. It is attractive because of the ease with which a scene can be modified. If the wires are pulled a different amount then the scene is modified almost instantly.

Most conventionally produced animation requires a piece of equipment called an animation stand to aid in converting static pieces of art into an animated film. "The animation stand is designed to provide a precise yet flexible relation between the camera and the artwork it is photographing." (Madsen, 1969) It is important to creating the effect of a character walking across a background. The cells containing the drawing of the various poses necessary to depict a walk cycle are sequentially positioned over the peg-bars which insure registration. These pegbars are moved across the static background during the filming process thereby creating the effect of the character walking across the background scene.

Historically, the animation stands have been controlled by a number of hand cranks that must be properly adjusted and then varied between film exposures to create the desired effect. Animators have now become aware of the great advances possible by utilizing computer control of their animation stands. Once a digital computer has been properly programmed and the necessary

interface systems developed then a scene utilizing an animation stand can become simple to program.

Al Stahl of Animated Productions, Inc., New York, one of the country's outstanding animation directors has been dreaming of such a system for years. He now has such a device that he helped design. Oxberry Corporation and other suppliers built one at a cost of over \$150,000. It incorporates a teleprinter, an automatic tape punch, an 8000 word digital computer, a camera control console and a highly complex animation stand. The animation stand is connected to the digital computer through an interface that enables the computer to control the animation stand movements. There is a system program that allows the animator to program the moves between the film camera and the artwork very precisely both in physical distances and in time. (Kallis, 1969)

A computer controlled animation stand greatly simplifies and speeds up the technique of making fast moving motion pictures from a limited number of static pictures or artwork. This animation technique is known by several names as Al Stahl calls the technique Fotomation and Roy Madsen calls it Filmograph. The pans, zooms, tilts and dissolves can make objects in the static pictures appear to move. Computer control makes the moves so smooth that the apparent movement can be very realistic. There are several other facilities that now possess computer controlled animation stands such as the National Film Board of Canada, The Optical House of New York and Animated Productions, Inc., New York.

Some examples have been given of various methods people have used to attempt to lower the cost of animation and thereby

make it applicable to a number of situations. Education, entertainment, and advertising can make good use of animation if only it will fit within the budget. Partial solutions have been developed but as yet nothing has been discussed that will make significant reduction in the amount of hand labor involved in producing animation.

### 3.2 Use of Computers in Animation

Computer animation is an emerging technology. Many different people have seen its tremendous potential for simplifying and thereby increasing the speed and lowering the cost of the animation process. Most of the efforts directed toward implementing the process of film animation by computer have been confined to using existing hardware. Much of the talent of the innovative pioneers has been directed toward utilizing hardware and software that had been developed to solve scientific problems or to keep records. That any successes at all were produced is a strong statement of their talent. However it finally became evident that a great improvement in efficiency and quality could result if computer technology were developed that was specifically directed toward image generation.

A number of inventions and developments were necessary before the technology of computer animation could itself be developed. The digital computer had a beginning back in 1822 when Babbage invented the first mechanical computing device. The next important step was the translation of mechanical to electronic methods. This was accomplished by the University of Pennsylvania who completed their Eniac in 1946.

An extremely important achievement for the field of

animation was the first marriage between an electronic computer and a cathode ray tube (CRT). (Lehman,1972) Ordvac performed this feat in 1952. There now existed a means for a computer to communicate visually with people. With a CRT for an output device and a proper interface program, the solution to a complex problem could be presented in a more meaningful manner. Not only are results easier to interpret in pictorial rather than numerical form but a hard copy of a picture usually contains more information than an equal size print out of numbers. This is an advantage if the aim is to record a lot of data efficiently. However it also means that a lot of data is required to draw a picture. This fact is then a handicap if the aim is to draw pictures rather than to store data.

Mathematicians, engineers and physicists programmed the large computers to solve their problems and then viewed the results on a CRT. A prime example of the effects that could be obtained in this manner was produced by the Los Alamos Scientific Laboratory, and depicted the flow of water under a sluice gate. This animated sequence which lasted 10 minutes required 500 million separate calculations and would have taken centuries to complete by any other means. (Halas,1970) It was a very efficient manner of demonstrating the data and is perhaps the only way to develop such a sequence. However, unless the accuracy of the method is required it is unnecessarily time consuming and therefore an expensive process.

Very soon scientists and mathematicians with an artistic bent saw the possibilities of creating an output whose purpose was not merely to demonstrate the result of a complex equation but

was also an artistic expression. Not surprisingly, the location at which much of the early work was done was the Bell Telephone Laboratories, Inc., in Murray Hill, N.J. E. E. Zajac developed much basic programming while producing a film "Simulation of a Two Gyro, Gravity-Gradient Attitude Control System." This film demonstrated the relationship between a satellite orbiting a rotating earth. It was a perfect example of a pictorial solution to a problem being much more meaningful and concise than the same results in the form of numbers. This film stimulated many people and led to more developments at Bell Labs.

Dr. Kenneth C. Knowlton of Bell Labs then produced a computer animated film titled "A Computer Technique for the Production of Animated Movies." In this film he demonstrated a new language BEFLIX, a language specifically dedicated to producing movies with computers. BEFLIX was a large step forward as it simplified the programming required to produce animation. It did have the disadvantage of requiring every point on the screen to be computed which is time consuming and therefore expensive. The advantage of this system is that little or no mathematical knowledge is required for programming. (Zajac, 1965)

William Fetter who was supervisor of the Computer Graphics Division of Boeing has been involved in the production of many different projects of great fascination. Perhaps his best known achievement was the programming of the "Boeing Man". This was a computer drawn man who was used in a number of films at Boeing, and also commercially for the Norelco Company. The man was used to study man-machine interactions and his measurements represent

the measurements of 50 percent of all Air Force pilots. (Wurman,1971)

Another achievement that these digital computer controlled displays can perform quite impressively is to simulate real life situations. William Fetter has produced a film of a simulated landing at an airport. This has also been accomplished by General Electric Company of Syracuse. Their system also is used as a moon landing simulator by the Apollo pilots. (General Electric,1973)

Evans and Sutherland Computer Corporation has simulated landing on an aircraft carrier for the Naval Training Device Center. The sequence is a complete flight, starting on the carrier deck taking off, flying in a circle and then landing on the carrier. (Sutherland,1970) Various degrees of realism can be achieved but technology exists to produce excellent simulation. All of the information about the subject to be simulated must be stored in the computer's memory. If landings at a particular airport are to be simulated then information about the airport itself must be stored. The amount of stored data determines the resolution that can be achieved. More detail requires more data. Then operating programs recall the pertinent data and use it to produce images that represent the actual scene.

Simulation, while very useful and exciting, is not animation or art. A number of people who saw the great potential of computer graphics were interested in creating art or beauty instead of answers to equations. Dr. Knowlton was an early pioneer in the production of films by computer and has written about the question of who are the artists in the new media. Are they people traditionally called artists, or computer programmers?

"These two groups are, I think, quite different bunches of people. I would describe both groups as creative, imaginative, intelligent, energetic, industrious, competitive, and driven. But programmers, in my experience- and myself included- tend to be constricted, painstaking, logical, precise, inhibited, cautious, restrained, defensive, methodical, ritualistic, cold and inscrutable in human terms. Their exterior actions are separated from their emotions by several layers of logical defenses, so they can always say "why" they did something, without in the least revealing their ultimate motives. Artists, in my experience, are free, alogical, intuitive, impulsive, implicit, confused, bewildered, bewildering, experimentive, perceptive, honest, frustrated, sensitive, and vulnerable. They do things without being able to say why they do them nor what they were trying to accomplish: the rules of the game are that one doesn't ask. In view of these considerations, I expect art to come from artists or artists working closely with programmers."

Dr. Knowlton has collaborated on several artistic films. "Pixillation" was produced by Maplewood, N.J. artist Lillian Schwartz and Dr. Knowlton and won first prize in the 1970 Eighth Annual Computer Art Contest. He also worked on "Man and His World", a 50 second film produced for EXPO '67 with Stan Vanderbeek, the experimental filmmaker. These films featured random and controlled complex patterns that would have been impossible to produce by conventional animation techniques. (Knowlton, 1970)

Another technical advance has been the advent of interactive computer systems. When operating this type of system the

operator artist can make changes in the program producing the imagery or control it while watching it. Professor Charles Cauri of Ohio State University uses a digital computer with a CRT output and light pen control to draw and control images. His "Hummingbird" which generates 14,000 modulations on one form is a unique production. (Davis,1971)

Also pioneering in the technique of interactive control is Dr. Ron Baecker. He recognized the value of immediate visual feedback. In conventional animation the artist must usually wait days to see the results of his drawings and check the effectiveness of his efforts. Even normal computer techniques usually require hours to complete the cycle between programming and playback. Dr. Baecker has developed a system implemented on the M.I.T. Lincoln Laboratory TX-2 computer called GENESYS. It is a picture-driven animation system allowing for rapid image programming and then control to produce a type of animation. (Baecker,1969) However, the imagery is quite crude, line drawings made up of a series of dots. Most of Dr. Baeckers proposals and suggestions for future work seem to be directed toward improved motion control.

This is consistent with other users of digital computers who seem to be more concerned with controlling motion than with the image quality of the output. It could be because this is the area where digital computers show their greatest promise. The movements of a character could be developed by computer but the actual imagery for final production would still have to be hand drawn to achieve satisfactory quality. Mr. Stan Hayward, a free-lance animator living in London, has worked with computer equipment



donated by Honeywell Information Services. Much of his work is done for Malas and Batchelor one of the leading animation studios in London. His use of the computer would encompass all areas of film making but in animation it would mostly assist in developing timing. He feels that the application of full computer animation in a commercial studio is limited to diagram work and certain special effects. "The difficulties of describing and manipulating complex non-geometrical shapes and the lack of full color reproduction in any form of computer animation clearly limit its possibilities as far as commercial animated films are concerned."

(Berenyi,1971)

Four people have been working on an interactive computer system at the University of Pennsylvania. They use several computers, a DEC-338, and IBM 360/75 and a S-D4020 microfilm recorder. The DEC-338 serves as the input terminal while the IBM 360/75 generates the proper instructions for the microfilm recorder. Once again their system seems capable of aiding in developing the timing of a hand drawn film. It also could produce some animated sequences that could illustrate some basic concepts for a teacher. However, the imagery possible is once again very limited and this fact severely limits the system's applications. (Talbot,1971)

An artist who contributed much to the new field of motion graphics is John Whitney. Mr. Whitney conceived the idea of producing abstract designs in motion. He started producing this new kind of film with the aid of instruments that he constructed from war-surplus analog devices, mostly weapon control devices. Dr. Jack Citron of IBM's Los Angeles Scientific Center saw one of

Mr. Whitney's films and became very interested in aiding in their production. He developed a basic program with 60 parameters that could be varied to control the image produced. A great assortment of shapes could be described by the program or algorithm. Some examples are circles, rosettes, conic sections, cylinders, hyperboloids and other curves as well as straight line and three-dimensional figures in perspective. Mr. Whitney became very adept with this program and supported by an IBM research grant he produced some very striking animated film. His tools were an IBM 360/75 and an IBM 2250 display unit. He used conventional film techniques to add color and to edit the images into finished sequence. Whitney's film "Permutations" was shown at the 1968 Lincoln Center Film Festival in New York City. (IBM,1969)

Mr. Whitney started his experimental film production with an analog rather than a digital device. There are other researchers in the field of computer animation that did not subscribe to the theory that digital computers make the best animators. Miura, Iwata and Tsuda of the Hitachi Company in Japan have reported about their technique of using a hybrid computer to draw pictures composed of curves. The curves are drawn by an analog computer on a cathode ray tube. A digital computer calculates the tangents at particular points of the curves so that the analog computer can reproduce them. The digital computer can also perform the interpolation necessary to proceed from one drawing to the next without the artist having to draw the intervening pictures. (Miura,1967)

Another man, Lee Harrison III, decided that a hybrid computer was best equipped to produce animation. He invented a

method of controlling voltages applied to an X-Y monitor that enabled the ensuing Lissajous patterns to produce unique images. The invention was implemented first in the early sixties with the current technology that required electronic tubes which limited the potential.

A second generation model was constructed in 1967 using the developing technology of integrated circuits. This model was a much more versatile realization of the invention and produced some very exciting animated images. The potential of this hybrid computer was investigated during the following year and various applications were developed.

Images could be programmed in the computer, named Animac, in three dimensions. Animac performed the calculations so that a two dimensional representation of the three dimensioned object was displayed on a CRT. Images were constructed through the control of position and movement of "bones" while additional shape control resulted from the shape of the "skin" surrounding the bones,

The animation was produced by controlling certain parameters of the bones and the skin. One unique method of controlling a humanoid character was by using voltages derived from potentiometers incorporated in an anthropomorphic harness. The "animator" merely performed the antics desired for the animated character and the voltages from the harness controlled the character and thereby produced the animation.

#### 4. CONTROLLING MOVEMENT BY MAPPING

##### 4.1 Concept

Many investigators continue to search for better methods of controlling computer generated images. These better methods usually take the form of a computer language. Many systems have been developed that can produce varied images that, viewed in succession, are perceived as animation. Most of these systems can make an image translate, rotate, change size and even have some of the separate elements comprising the image move with respect to other elements of the image. Dr. DeFanti of the Ohio State University discusses a number of these languages in a report titled Real-Time Film Animation. (DeFanti,1973) The sources he cites are all searching for a universal means of controlling or producing animation with their systems.

A concept that can be very powerful in computer animation is that of mapping surfaces. To understand the application of this concept think of an artist's sketch pad as consisting of thin transparent rubber sheets. These sheets are the size of a dot in their normal state but they can be stretched to be any arbitrary size. An artist can draw on one of these sheets and then stretch or shrink them in any direction, have them rotate, bend, distort, change to a mirror image or have selected regions move.

Someone watching the artist's drawing on the rubber sheet while these changes are taking place would see a moving image which is defined as animation. If the artist merely wished to make a wheel appear to rotate he could first stretch his rubber sheet to an appropriate size. Then he could draw a picture of

the type of wheel he wished to rotate, an old wagon wheel with hub and spokes or a race car wheel with a racing hub and wide tire, on the sheet. Then by making the sheet rotate about the center of the wheel the artist would produce an animated wheel. Only the one picture of the wheel would be necessary for a long sequence of a rotating wheel. The same drawing could be used to depict a wheel coming from a distance, where it appears as a dot, until it is full screen or larger. In this case the sheet would slowly be stretched both horizontally and vertically so that the image on the sheet would appear to zoom. It could be rotating during the zoom and also moving up, down and sideways. Once again only one drawing would be necessary for any length sequence and the animated image, the wheel, would have the shape and detail that the artist put into his drawing.

One advantage of a mapping scheme over other computer animation systems is the manner in which the artist communicates his wishes to the computer. Many systems claim to require only ideas that exist in the artist's mind to produce animation. However, the computer cannot read a person's mind. In some manner the details of the imagined scene must be communicated to the computer as it can only work with the data given to it. If the artist is also a programmer he might be able to create the punched cards, magnetic tape or by some other means of digital communication, program the image in the computer. Generally, artists are not programmers and express their ideas best in pictures as their training dictates.

When an artist draws a logo, some graphics or a figure -

that is the image he wishes to animate. Most previously described systems require a technician or programmer to extract the details of the image and arrange them into a proper form for the computer. Even then it can be very difficult to communicate all of the important characteristics of the image as the programmer might not appreciate the fine points of the artist's drawings. An artist may not draw in a manner that is easily transferred by mathematical equations. If the image cannot be described by equations then a point by point description may be necessary. This type of interface between the artist and the computer is tedious at best and then may not be satisfactory.

Since the artist usually will draw his desired image anyway to describe it to the programmer, then it is advantageous to use his drawing directly to program the image. The next problem then is how is that image to be animated. This is where the concept of mapping enters. Mapping is the process by which the surface containing the artist's drawing can be transformed into another surface containing the same drawing with different characteristics. Its position, angular orientation, shape and size can be different from that of the initial surface thereby changing the characteristics of the drawing.

The display plane for the output image can be related to the initial or input plane by a mapping function. As long as the mapping function remains a constant, there is no animation. However, a carefully controlled series of mapping functions applied to the input plane can produce a series of output images that are an animated version of the input artwork. That is, a

mapping function that causes the input plane to translate can produce animation if the function varies as a result of time. The result could be an image moving from the left to the right, down or up or in any other conceivable manner desired.

This brings up another advantage, the amount of data necessary to specify the animation. Consider the simplest situation: that the output of the animation system be viewed on a television monitor. This requires less resolution than other means of display. Even so, intensity and color information for about 250,000 different points will need to be specified for each frame. (Sutherland, 1970) A minimum of six bits per point is necessary to identify the color intensity for each individual point. The total then adds up to  $1.5 \times 10^6$  bits that must be calculated for each frame produced by conventional computer techniques.

It takes much less data to control the mapping operator parameter values that can produce the animation by manipulating two dimensional planes containing static images. If the animation consists of ten planes moving in concert then the data required for control is approximately:

$$\frac{10 \text{ planes}}{\text{frame}} \times \frac{30 \text{ parameter values}}{\text{plane}} \times \frac{16 \text{ bits}}{\text{parameter value}} = \frac{4800 \text{ bits}}{\text{picture}} \quad (4.1)$$

The mapping technique requires only about 0.3% of the information necessary for the conventional methods of computer animation.

#### 4.2 Developing the Basic Mapping Operators

The key to the concept of producing animation by mapping input planes to output planes is control. Animation is

a refined art or even a science. An experienced animator can create the illusion of almost any desired type of movement by the types of images he presents to the viewer. Therefore, some method of controlling the type of mapping must be devised. Such a method is described.

As stated, this method of producing animation is essentially a mapping process. The input plane, the artwork, is mapped into the output plane through some function. Three different mapping functions are sufficient to produce a wide variety of animation. They are the rotation, translation and magnifying functions.

A convenient method of describing a mapping function that performs a rotation is to operate on the input plane with a rotation operator. Let the input plane  $\bar{Z}$  be described by the matrix

$$\bar{Z} = \begin{bmatrix} x \\ y \end{bmatrix} \quad (4.2)$$

since  $\bar{Z}$  is a two dimensional plane. Then the rotation operator could be described by the matrix

$$\bar{R} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \quad (4.3)$$

Using conventional matrix multiplication the effect of  $\bar{R}$  on  $\bar{Z}$  can be seen to be  $\bar{W}$ .

$$\bar{R}\bar{Z} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x\cos\theta - y\sin\theta \\ x\sin\theta + y\cos\theta \end{bmatrix} = \bar{W}_1 = \begin{bmatrix} u \\ v \end{bmatrix} \quad (4.4)$$

The rotated plane  $\bar{W}$  has as its horizontal component  $u=y\cos\theta-x\sin\theta$  and as its vertical component  $v=x\sin\theta+y\cos\theta$ . A line that was horizontal on the input plane,  $\bar{Z}$ , that is its angular displacement was 0 radians, will now be displaced by  $\theta$  radians from the horizontal. This is because the display raster, plane  $W$ , was rotated around the  $\bar{Z}$  axis by the angle,  $\theta$  radians.



The translation of a plane by a vector

$$\bar{T} = \begin{bmatrix} t1 \\ t2 \end{bmatrix}$$

can also be described by matrix manipulation. This mapping function is additive and the result is shown by the equation

$$\bar{W}_2 = \bar{T} + \bar{Z} = \begin{bmatrix} t1 \\ t2 \end{bmatrix} + \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} t1+x \\ t2+y \end{bmatrix} \quad (4.5)$$

The result shows that x is shifted by an amount equal to t1 and that y is shifted by an amount equal to t2. If the input plane,  $\bar{Z}$ , had a symbol of some kind on it then it would appear to be shifted by an amount t1 horizontally and by an amount t2 vertically on the display raster plane,  $\bar{W}$ . Again this is because the display raster or plane was translated by the vector  $\bar{T}$  to be located in a different position by the translation operator.

The other important mapping function is that of the magnifier. The magnifying mapping function is a multiplicative one as was the rotation function. Let the magnifying operator be the matrix

$$M = \begin{bmatrix} m1 & 0 \\ 0 & m2 \end{bmatrix} \quad (4.6)$$

where m1 is the factor that will control the x dimension and m2 will control the y dimension. The effect can again be shown by matrix multiplication.

$$\bar{W}_3 = M\bar{Z} = \begin{bmatrix} m1 & 0 \\ 0 & m2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} m1x \\ m2y \end{bmatrix} \quad (4.7)$$

As was desired, the x dimension is magnified by the factor m1 and the y dimension is magnified by the factor m2. The visible effect is that of changing the size and or the aspect ratio of the input artwork. For a simple zoom effect, m1 equals m2.

It is possible to describe the necessary mapping function by matrix manipulations. However, two different types of manipulations, multiplication and addition, are necessary. It is convenient to be able to describe these mappings by the action of a single type of operator.

Such an operator could result if the two-dimensional plane is referred to as

$$\bar{z} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \quad (4.8)$$

in matrix notation where the third row is always unity. Any operator then can be represented by a 3x3 matrix. Let  $\bar{A}$  represent a general operator. In matrix notation this is

$$\bar{A} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \quad (4.9)$$

The last row is always 0, 0, 1 so that when the operator  $\bar{A}$  transforms plane  $\bar{z}$  as determined by conventional matrix multiplication, the last row of the resulting 3x1 matrix is unity. To demonstrate:

$$\bar{A}\bar{z} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} ax + by + c \\ dx + ey + f \\ 1 \end{bmatrix} \quad (4.10)$$

It can be seen that "a" is the multiplicand of the horizontal factor of the input plane in the horizontal factor of the output plane. "b" is the multiplicand of the vertical factor of the input plane in the horizontal factor of the output plane. "c" is the horizontal translation of the output plane relative to the input plane. "d" is the multiplicand of the horizontal factor of the input plane in the vertical factor of the output plane. "e" is the multiplicand of the vertical factor of the input plane in the vertical factor of the output plane. "f" is the vertical

translation of the output plane relative to the input plane.

If the operator is to be multiplicative then  $c = f = 0$  with  $a, b, d$  and  $e$  having appropriate values. An additive operator has  $a = e = 1, b = d = 0$  with  $c$  and  $f$  having appropriate values.

The three essential mapping functions; rotation, magnification and translation; can be determined in this format. A rotation operator  $R[p]$  may be developed where  $p$  is the angle through which the plane is rotated. The desired result, the plane  $\bar{Z}$  rotated through the angle  $p$ , is the plane

$$\bar{R}[p]\bar{Z} = \begin{bmatrix} x \cos p - y \sin p \\ x \sin p + y \cos p \\ 1 \end{bmatrix} \quad (4.11)$$

From this matrix the specific elements of the general matrix  $A$  can be found. The proper result is obtained when  $a = e = \cos p$ ,  $d = \sin p$ ,  $b = -\sin p$  and  $c = f = 0$ . Therefore,

$$\bar{R}[p] = \begin{bmatrix} \cos p & -\sin p & 0 \\ \sin p & \cos p & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (4.12)$$

with  $c = f = 0$  as befits a multiplicative operator.

Another operator that can be developed is the magnifying operator,  $M[B1, B2]$ , where  $B1$  is the magnifying factor of the horizontal axis and  $B2$  is the magnifying factor of the vertical axis. The result of operating on the plane  $Z$  with the magnifying operator,  $M[B1, B2]$ , should be

$$\bar{M}[B1, B2]\bar{Z} = \begin{bmatrix} B1x \\ B2y \\ 1 \end{bmatrix} \quad (4.13)$$

This could be accomplished by letting  $a$  and  $e$  of the general operator,  $A$ , equal  $B1$  and  $B2$  respectively and the other terms,  $b, c, d$  and  $f$ , equal 0. Then

$$\bar{M}[B1, B2]\bar{Z} = \begin{bmatrix} B1 & 0 & 0 \\ 0 & B2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} B1x \\ B2y \\ 1 \end{bmatrix} \quad (4.14)$$

which is the desired result. Again the fact that  $c = f = 0$  indicates that  $M[B1, B2]$  is a multiplicative operator.

Still another necessary operator is one that can produce translations. The plane  $\bar{z}$  translated by a factor of  $C1$  in the horizontal direction and  $C2$  in the vertical direction could be represented by the plane

$$\bar{T}[C1, C2]\bar{z} = \begin{bmatrix} x+C1 \\ y+C2 \\ 1 \end{bmatrix} \quad (4.15)$$

This could be accomplished by setting  $a = e = 1$ ,  $b = d = 0$ ,  $c = C1$  and  $f = C2$ . Then

$$\bar{T}[C1, C2]\bar{z} = \begin{bmatrix} 1 & 0 & C1 \\ 0 & 1 & C2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x+C1 \\ y+C2 \\ 1 \end{bmatrix} \quad (4.16)$$

which is the desired result. The facts that  $c$  and  $f \neq 0$  while  $a = e = 1$  and  $b = d = 0$  indicate that the translation operator,  $\bar{T}[C1, C2]$ , is an additive operator.

Therefore it can be concluded that by representing a two-dimensional plane by the vector

$$\bar{z} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \quad (4.17)$$

which is a  $3 \times 1$  matrix, a single form of operator can be used to represent either a multiplicative or an additive operator. This form is a  $3 \times 3$  matrix whose last row is always  $0, 0, 1$ . The other elements take different values depending on the type of operator.

It can be shown that additive operators are commutative with other additive operators and multiplicative operators are commutative with other multiplicative operators. However, the two types of operators are not commutative when intermingled. The two types of operators are associative in any order or combination.

The different transformations or mapping functions may be either conformal or non-conformal. A theorem from Churchill (1960) states that; "At each point  $z$  of a domain where  $f$  is analytic and  $f'(z) \neq 0$  the mapping  $w=f(z)$  is conformal." Also, he writes that "every conformal transformation must map orthogonal curves into orthogonal curves." Therefore a transformation preserves angles.

A basic conformal mapping function is the general linear function  $w = Bz + C$  where  $w$  is the output plane,  $z$  is the input plane and  $B$  and  $C$  are vectors. The effect of the vector  $B$  is to magnify  $z$  by the absolute value of  $B$  and to rotate  $z$  through an angle equal to the argument of  $B$ . The effect of the vector  $C$  is to translate  $z$  in the horizontal direction by a factor equal to the real part of  $C$ ,  $C_1$ , and in the vertical direction by a factor equal to the imaginary part of  $C$ ,  $C_2$ .

The same transformation could be written in terms of the operators just developed. The expression

$$w = T(C_1, C_2)M(\beta, \beta)R(\alpha)z \quad (4.18)$$

where  $B = \beta e^{i\alpha}$  leads to

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & C_1 \\ 0 & 1 & C_2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \beta & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \quad (4.19)$$

or

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \beta(x\cos\alpha - y\sin\alpha) + C_1 \\ \beta(x\sin\alpha + y\cos\alpha) + C_2 \\ 1 \end{bmatrix} \quad (4.20)$$

which is identical to the result produced by the expression

$$w = Bz + C \quad (4.21)$$

The three mapping functions, as described, will produce certain changes of the artwork on the display with reference to the input artwork. It can be rotated through any angle and translated any amount in any direction. It can be zoomed so that its size can vary from a point to a size that is greater than the input artwork. The aspect ratio can also be varied so as to make the displayed artwork short and fat or tall and thin. These apparent artwork manipulations are accomplished with mapping functions whose arguments are constant during the display of a single frame or picture.

#### 4.3 Shaping Images with the Mapping Operators

A very interesting and useful type of image manipulation results when the arguments are a function of the display plane or some dimension of the display plane. Let the translation function serve as an example. A translation mapping with constant arguments was written as

$$\bar{T}[C1, C2] \bar{Z} = \begin{bmatrix} 1 & 0 & C1 \\ 0 & 1 & C2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = \begin{bmatrix} X+C1 \\ Y+C2 \\ 1 \end{bmatrix} \quad (4.22)$$

Now to achieve a bending effect let the vertical translation be a function of the horizontal position. Then C2 can be written as

$$C2 = A \sin(k_1 x) \quad (4.23)$$

where A is the amplitude and the value of  $k_1$  determines the number of cycles across the width of the display plane. The translation mapping then takes the form

$$\bar{T}[0, A \sin(k_1 x)] \bar{Z} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & A \sin(k_1 x) \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = \begin{bmatrix} X \\ Y + A \sin(k_1 x) \\ 1 \end{bmatrix} \quad (4.24)$$

If the value of  $k_1$  is such that there is one cycle across the

width of the plane, that is values of  $\sin(k_1 x)$  range from 0 at  $x=0$ , to 1 at  $x=\frac{\pi}{2k_1}$ , to 0 at  $x=\frac{\pi}{k_1}$ , to -1 at  $x=-\frac{\pi}{2k_1}$  and finally to 0 at  $x=-\frac{\pi}{k_1}$  then the boundaries of the display plane will have the shape of a sine wave. Other possibilities are shown in Figure 4.1. The input artwork will be comparably effected when viewed on the display.

The same type of effect can also be applied to the other axis. This time let

$$C_1 = B\cos(k_2 y) \quad (4.25)$$

Then the mapping of the input plane through a translation can be written as

$$\bar{W} = \bar{T}[B\cos(k_2 y), 0] \bar{Z} \quad (4.26)$$

$$\bar{W} = \begin{bmatrix} 1 & 0 & B\cos(k_2 y) \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = \begin{bmatrix} X+B\cos(k_2 y) \\ Y \\ 1 \end{bmatrix} \quad (4.27)$$

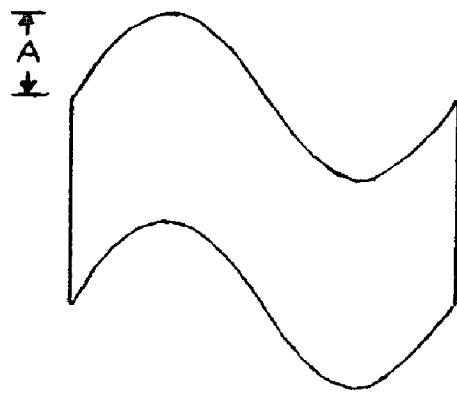
where as before,  $B$  is the amplitude of cosine wave and  $k_2$  is the factor that determines the number of cycles present in the plane. Examples of distortions possible by this effect are shown in Figure 4.2.

Both effects may be present at the same time. To obtain a very unusual image let

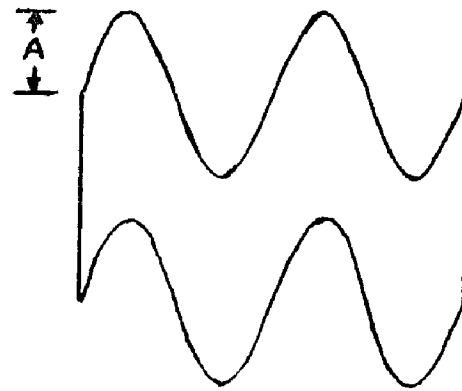
$$C_1 = B\cos(k_2 y); \quad C_2 = A\sin(k_1 x) \quad (4.28)$$

Then the output plane with respect to the input plane can be found from

$$\bar{W} = \bar{T}[B\cos(k_2 y), A\sin(k_1 x)] \bar{Z} = \begin{bmatrix} X+B\cos(k_2 y) \\ Y+A\sin(k_1 x) \\ 1 \end{bmatrix} \quad (4.29)$$



(a) One Cycle



(b) Two Cycles

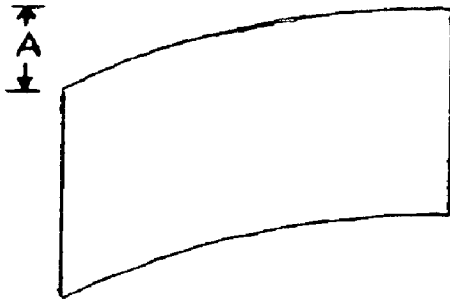
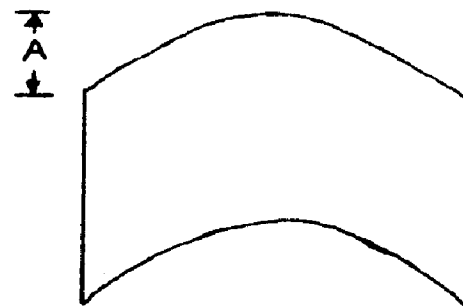
(c)  $\frac{1}{4}$  Cycle(d)  $\frac{1}{2}$  Cycle

Figure 4.1 Shaping Effect on Rectangular Planes of Vertical Translation Functions Which are Sine Waves With Different Periods



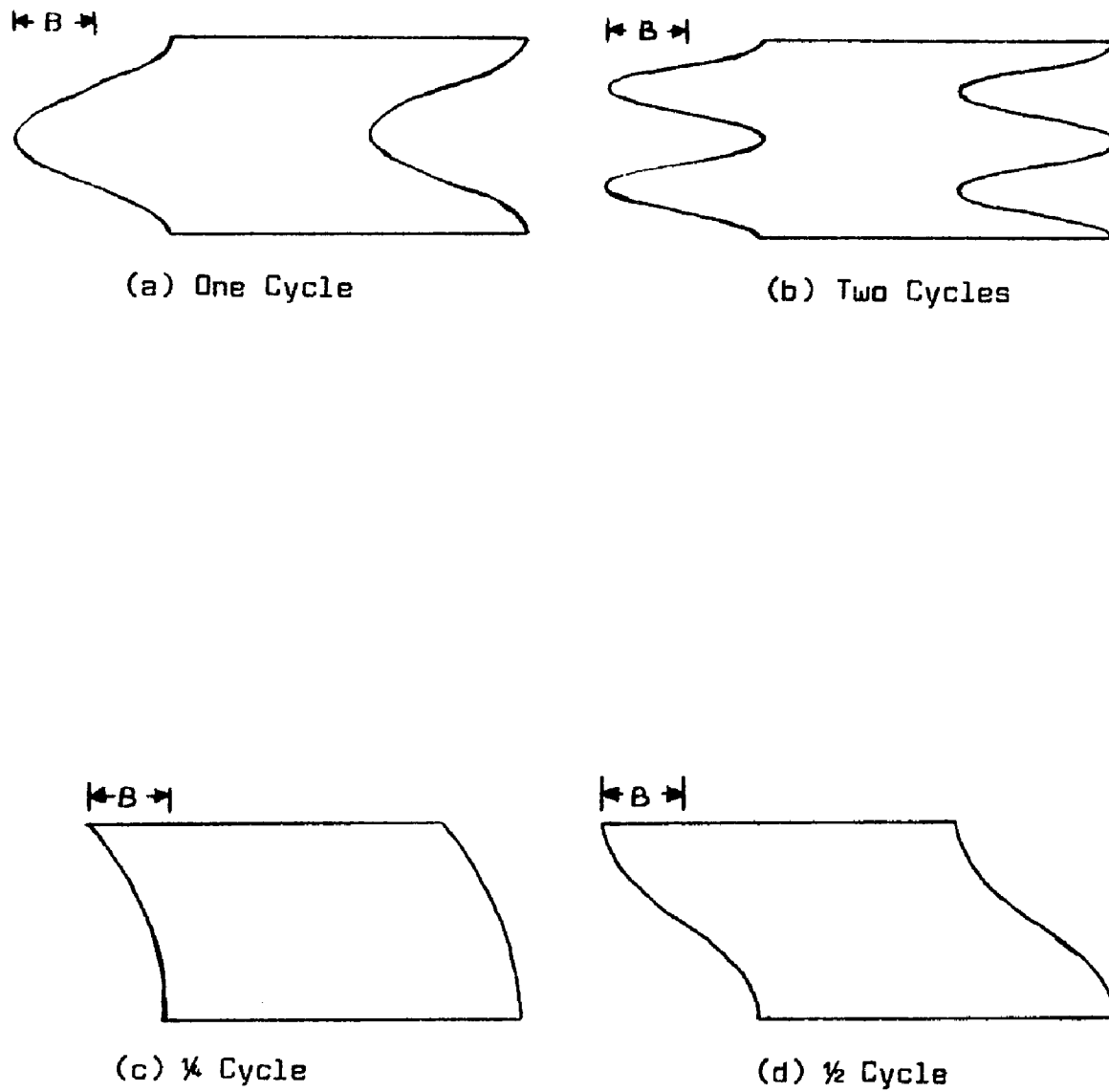


Figure 4.2 Shaping Effect on Rectangular Planes of Horizontal Translation Functions Which are Cosine Waves With Different Periods

These effects make it possible to vary the shape of the input artwork. The examples that are presented made use of sine or cosine signals as the modulation signal. However the signal may be a triangle wave, a square wave or some completely arbitrary waveform. When both vertical and horizontal modulations are present at the same time the effect created can be a type of abstract. If the image of an arm is to be bent at the elbow then a vertical translation is used. The amount of the bend is controlled by the amplitude of the translation function employing a triangle waveform as shown in Figure 4.3.

Earlier examples of mapping functions had arguments that were constant over an entire frame. Now mapping functions whose arguments are functions of position have been introduced. There is a restriction on the amount of bending that can be produced by this type of translation mapping before the output image loses the character of the input image. For example, a too large amplitude of the vertical translation that produces arm bending will distort the arm until the output image is unrecognizable as an arm.

Letting the argument of the mapping function be a function of position can also be used in the magnifying operator. For an example, let the horizontal magnifier,  $B_1$ , be a function of the horizontal position. If

$$B_1 = A+Bx \quad (4.30)$$

then the operation is;

$$\overline{M}[A+Bx, A] \overline{Z} = \begin{bmatrix} A+Bx & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = \begin{bmatrix} Ax+Bx^2 \\ AY \\ 1 \end{bmatrix} \quad (4.31)$$

The result shows that if  $B = 0$  then the entire plan is just magni-

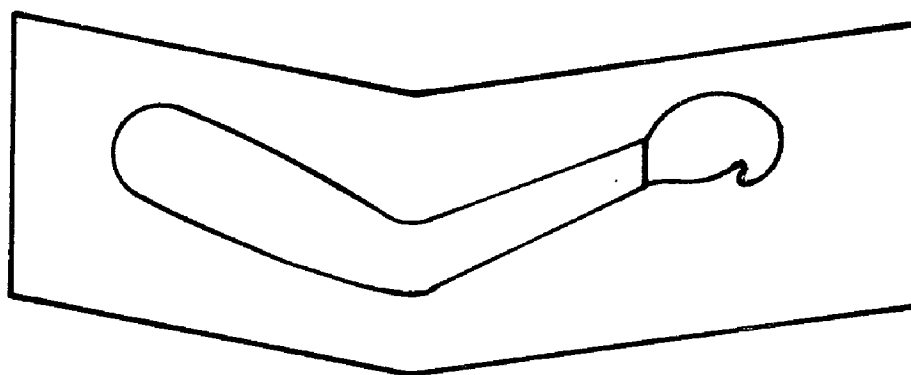
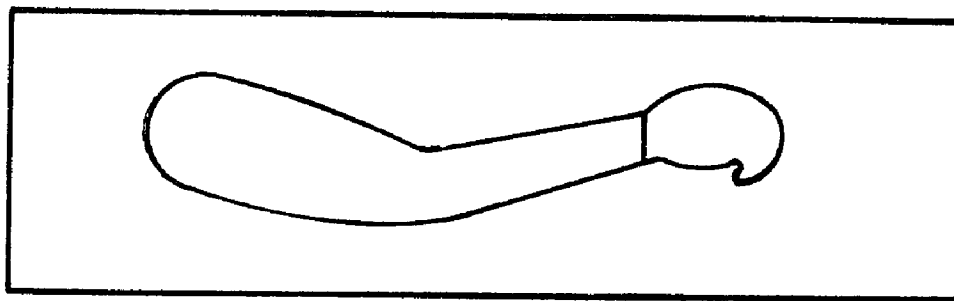


Figure 4.3 Bending an Arm by Employing a Triangle Waveform in a Translation Operator

fied by the factor A. If the value of B is not zero then the horizontal dimension of the plane is stretched as the value of the horizontal position is increased. If the B is a function of time, like

$$B = \sin(\omega_1 t) \quad (4.32)$$

then the horizontal dimension of the output plane would stretch and shrink at a frequency of  $\omega_1$  radians/second.

The mapping functions whose arguments were constant over an entire frame were conformal mapping functions. Briefly the functions are

$$\bar{w} = f(\bar{z}) \quad (4.33)$$

Magnification;

$$\bar{w} = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \bar{M}[a, b] \bar{z} = \begin{bmatrix} ax \\ by \\ 1 \end{bmatrix} \quad (4.34)$$

Translation;

$$\bar{w} = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \bar{T}[C1, C2] \bar{z} = \begin{bmatrix} X+C1 \\ Y+C2 \\ 1 \end{bmatrix} \quad (4.35)$$

Rotation;

$$\bar{w} = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \bar{R}[\theta] \bar{z} = \begin{bmatrix} x \cos \theta - y \sin \theta \\ x \sin \theta + y \cos \theta \\ 1 \end{bmatrix} \quad (4.36)$$

Each of these functions can be tested by the Cauchy-Riemann equations to see if they are analytic and therefore conformal mapping operators. The Cauchy-Riemann equations are

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad (4.37)$$

and

$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \quad (4.38)$$

For operator (4.33)

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} = a \text{ if } a=b \quad (4.39)$$

$$\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x} = 0 \quad (4.40)$$

For operator (4.34)

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} = 1 \quad (4.41)$$

$$u = v = 0 \text{ if } C1 \neq f(y) \text{ and } C2 \neq f(x) \quad (4.42)$$

For operator (4.35)

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} = \cos\theta \quad (4.42)$$

$$\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x} = \sin\theta \quad (4.43)$$

When "a" does not equal "b" then the magnification operator is not an analytic function and thereby is non-conformal. A similar circumstance exists for the translation operator when C1 is a function of y or when C2 is a function of x. Both of these situations can easily be seen to be non-conformal as angles are not preserved through the mapping process. When the aspect ratio is varied or the plane is distorted by a bending translation then angles on the plane are changed.

#### 4.4 Moving Images With Time Varying Operators

The property of making the amplitudes of the arguments or the variables of the arguments be arbitrary functions of time is what makes these mapping processes capable of producing animation. Referring to a single frame or to a single point in time the mapping process will only produce a static image that is

arbitrarily different from the input artwork. It may be a different size, be positioned differently, rotated with respect to, be bent, warped or distorted beyond recognition but it is only a static picture. If the arguments of the magnifying operator are functions of time like

$$B_1 = B_2 = Gt \quad (4.44)$$

then

$$\bar{M}[Gt, Gt] \bar{Z} = \begin{bmatrix} Gtx \\ Gty \\ 1 \end{bmatrix} \quad (4.45)$$

This result states that the image starts as a point at time  $t = 0$ , grows to size  $G$  times the input artwork at time  $t = 1$  and continues to grow in size as time increases. This produces the effect known in animation as a zoom. Another example may be to let the argument  $p$  of the rotation operator be a function of time such as

$$p = \omega_1 t \quad (4.46)$$

Then

$$\bar{R}[\omega_1 t] \bar{Z} = \begin{bmatrix} x \cos(\omega_1 t) - y \sin(\omega_1 t) \\ x \sin(\omega_1 t) + y \cos(\omega_1 t) \\ 1 \end{bmatrix} \quad (4.47)$$

This result describes the display plane rotating at a frequency of  $\omega_1$  radians per second and describes an animation of the input artwork.

Suppose it is desired to make an arm appear to knock at a door. The arm should appear to bend at the elbow as the arm prepares to strike out. Then the arm straightens as the fist hits the door's surface. Therefore the time sequence can be divided into

two parts. During the first period the bending translation function has an increasing amplitude while at the same time the horizontal size is decreasing. It is necessary that these two operators, translation for the bend and magnifying for the horizontal size change, be dependent on the same time function in order to keep the length of the arm a constant. If the horizontal size is not decreased proportionately as the amount of bend is increased then the arm would be stretched at the same time that it was bent. That is,  $K'$  is greater than  $K$  as shown in Figure 4.4. The second part of the sequence has the arm straightening as the knuckles strike the door. Therefore the amplitude of the bending translation operator goes to zero and the horizontal size returns to its nominal value.

Actually the effect produced by the events just described is that of a straight arm bending and then straightening again. A more complicated sequence is desirable to produce the image of an arm knocking at a door. The arm would probably start in a vertical position and rotate to a horizontal position, while at the same time bending. Also, the time required for the arm to rotate while contracting and bending would probably be greater than the time required for the arm to straighten and strike the door. As an example, let the time taken by the first part of the sequence be  $T_1$  and the time taken by the second part of the sequence be  $T_2$ .

The first part of the segment, where time varies between  $t = 0$  and  $t = T_1$ , can be described by the equation

$$\bar{W}_1 = \bar{R} \left[ \frac{\pi}{2} \left( \frac{t}{T_1} - 1 \right) \right] \bar{T} \left[ x_1, y_1 + \frac{t A_1}{T_1} V(x) \right] \bar{M} \left[ 1 - \frac{t m}{T_1}, 1 \right] \bar{Z} \quad (4.48)$$

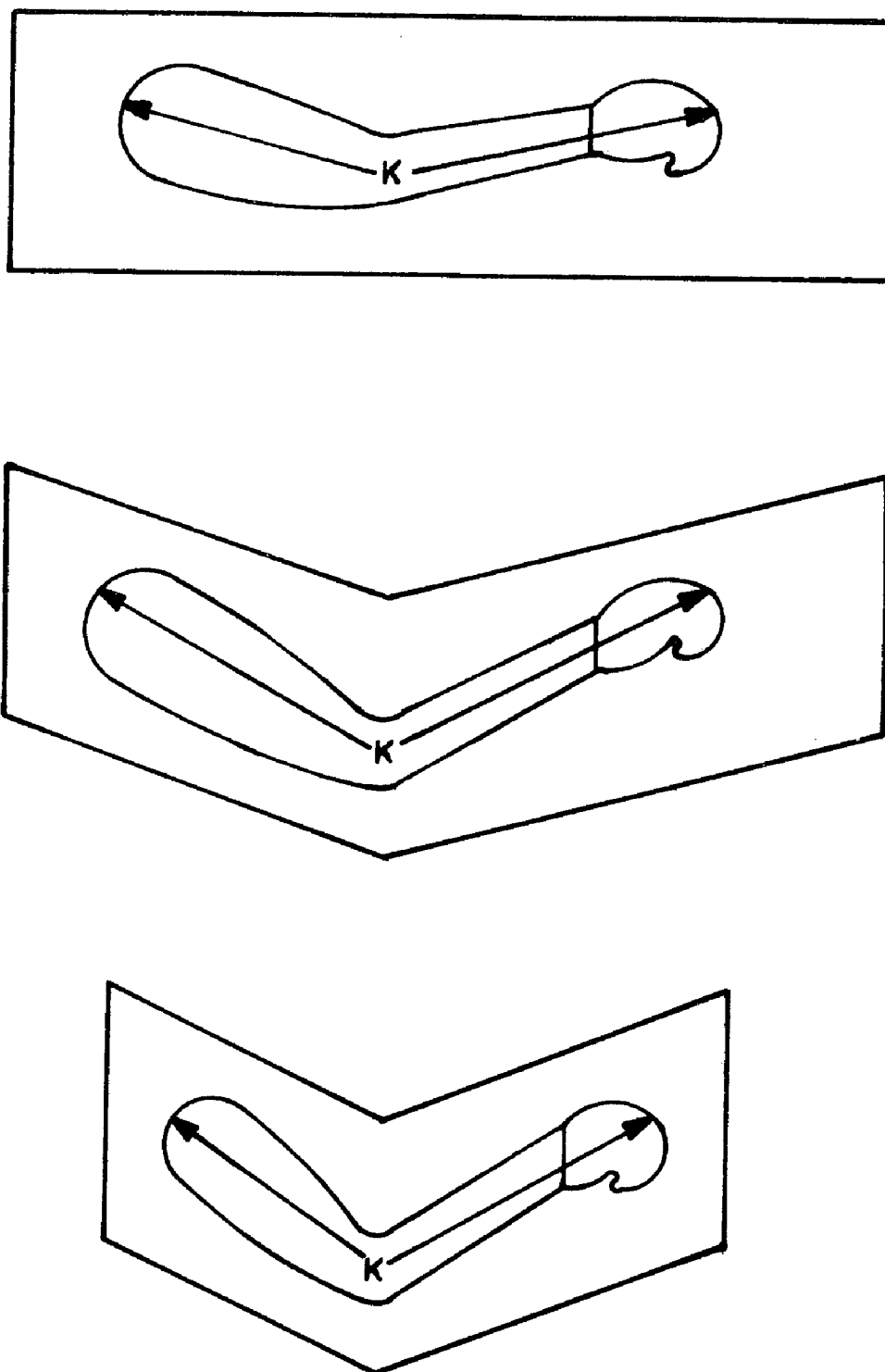


Figure 4.4 Effect of Horizontal Size on Maintaining Proper Perspective During Bending



where  $A_1$  is the desired maximum amplitude of the triangle wave,  $V(x)$ , that produces the required bending. In like manner the second part of the segment, where time varies between  $t = T_1$  and  $t = T_2$ , can be described by the equation

$$\bar{W}_2 = \bar{R}[0] \bar{T}\left[0, \frac{(T_2-t)}{T_2-T_1} A_1 V(x)\right] \bar{M}\left[1 - \frac{(T_2-t)}{T_2-T_1} m_1, 1\right] \bar{Z} \quad (4.49)$$

Here  $m_1$  is the percentage of nominal by which the width of the plane must be shortened to keep the arm length constant. By utilizing unity step functions the entire two step sequence can be described by a single equation:

$$\begin{aligned} \bar{W} = & [h(t) - h(t-T_1)] \bar{R}\left[\frac{\pi}{2}\left(\frac{t}{T_1} - 1\right)\right] \bar{T}\left[x_1, y_1 + \frac{tA_1}{T_1} V(x)\right] \\ & \bar{M}\left[1 - \frac{tm_1}{T_1}, 1\right] \bar{Z} + [h(t-T_1) - h(t-T_2)] \bar{R}[0] \\ & \bar{T}\left[0, \frac{T_2-t}{T_2-T_1} A_1 V(x)\right] \bar{M}\left[1 - \frac{T_2-t}{T_2-T_1} m_1, 1\right] \bar{Z} \end{aligned} \quad (4.50)$$

where  $h(t)$  is the Heaviside unit function.

The Heaviside unit function, hereafter referred to as a step function, that has the form  $h(t-T_1) - h(t-T_2)$  limits operations to the time interval,  $T_1 \leq t \leq T_2$ , (Kaplan, 1962). Therefore the equation representing the output plane  $\bar{W}$  consists of two sets of operations. The first set is active during the time interval  $0 \leq t \leq T_1$  and the second is active during the time interval  $T_1 \leq t \leq T_2$ . The equation states that during the first time interval the length of the arm is shortened as a function of time varying from the nominal plane width at  $t = 0$  to a plane width of  $(1-m_1)$  of nominal at  $t = T_1$ . At the same time the amplitude of the triangular bending function varies from zero at  $t = 0$  to  $A_1$  at  $t = T_1$ . Also the center of rotation is set to the point  $X = X_1$  and  $Y = Y_1$ . Finally the arm is rotated about the point  $(-X_1, -Y_1)$

from an initial angle of  $-\frac{\pi}{2}$  at  $t=0$  to an angle of 0 radians at  $t = T_1$ . During the time interval  $T_1 \leq t \leq T_2$  the width of the output plane increases from a value of  $(1-m_1)$  of nominal at  $t = T_1$  to a value of 1 at  $t = T_2$ . Also the amplitude of the triangular bending function changes from the value of  $A_1$  at  $t = T_1$  to the value of 0 at  $t = T_2$ . The shape of the arm at specific moments during the action is shown in Figure 4.5.

A series of movements or animations can be described by a single equation that relates the output plane  $\bar{W}$  to the input plane  $\bar{Z}$ . The various Heaviside functions would determine the time interval required for each member of the series. Each of the animations could require various combinations of the possible mapping functions. Therefore, it is possible that the equation could be written describing a sequence of the arm rotating from the vertical while bending and then stretching out to hit a surface, withdrawing to the bent position and then again striking the surface three times in succession. Assuming that the sequence ends when the fingers strike the door the third time, the series of animation would have six members. The key frames or poses would look like Figure 4.6.

It can be seen then that the output plane  $\bar{W}$  is actually a function of time. During a sequence the plane assumes many different sizes, shapes and orientations; all of them specified by the arguments of the different mapping functions.

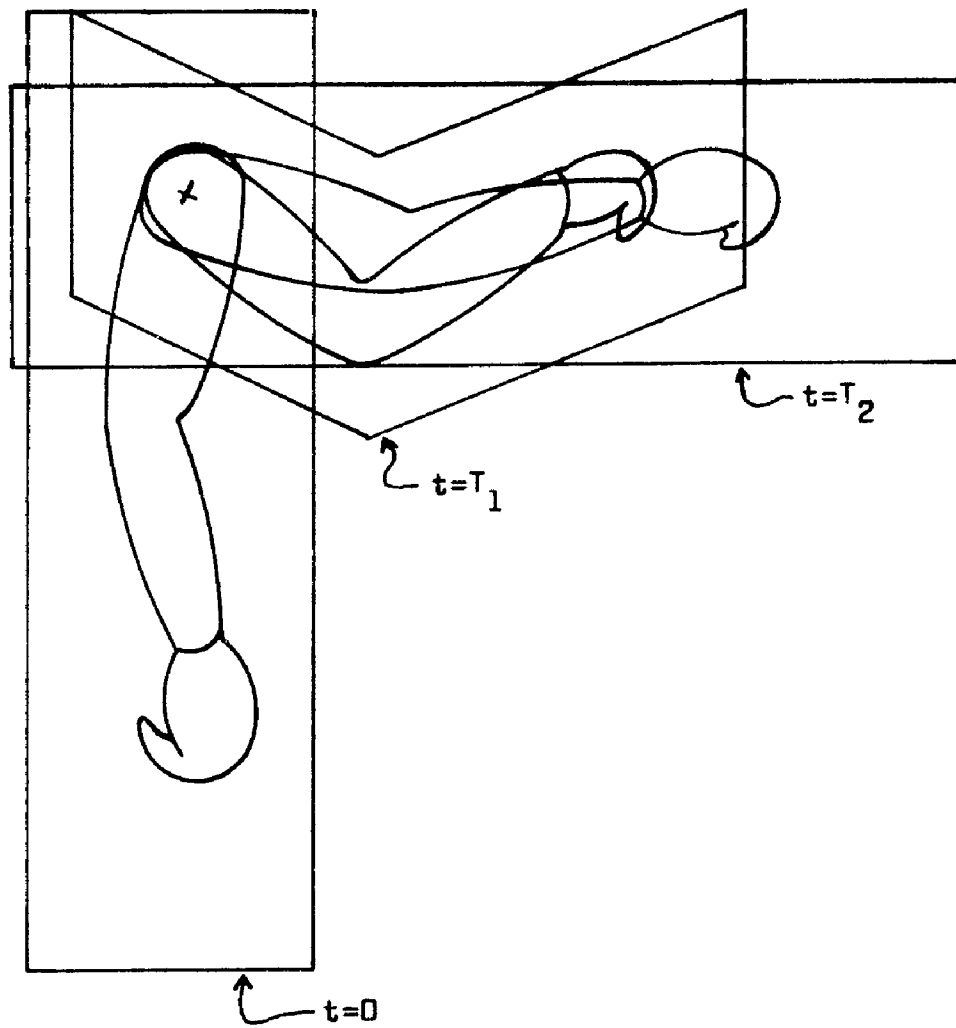


Figure 4.5 Three Key Frames for a "Knocking" Sequence

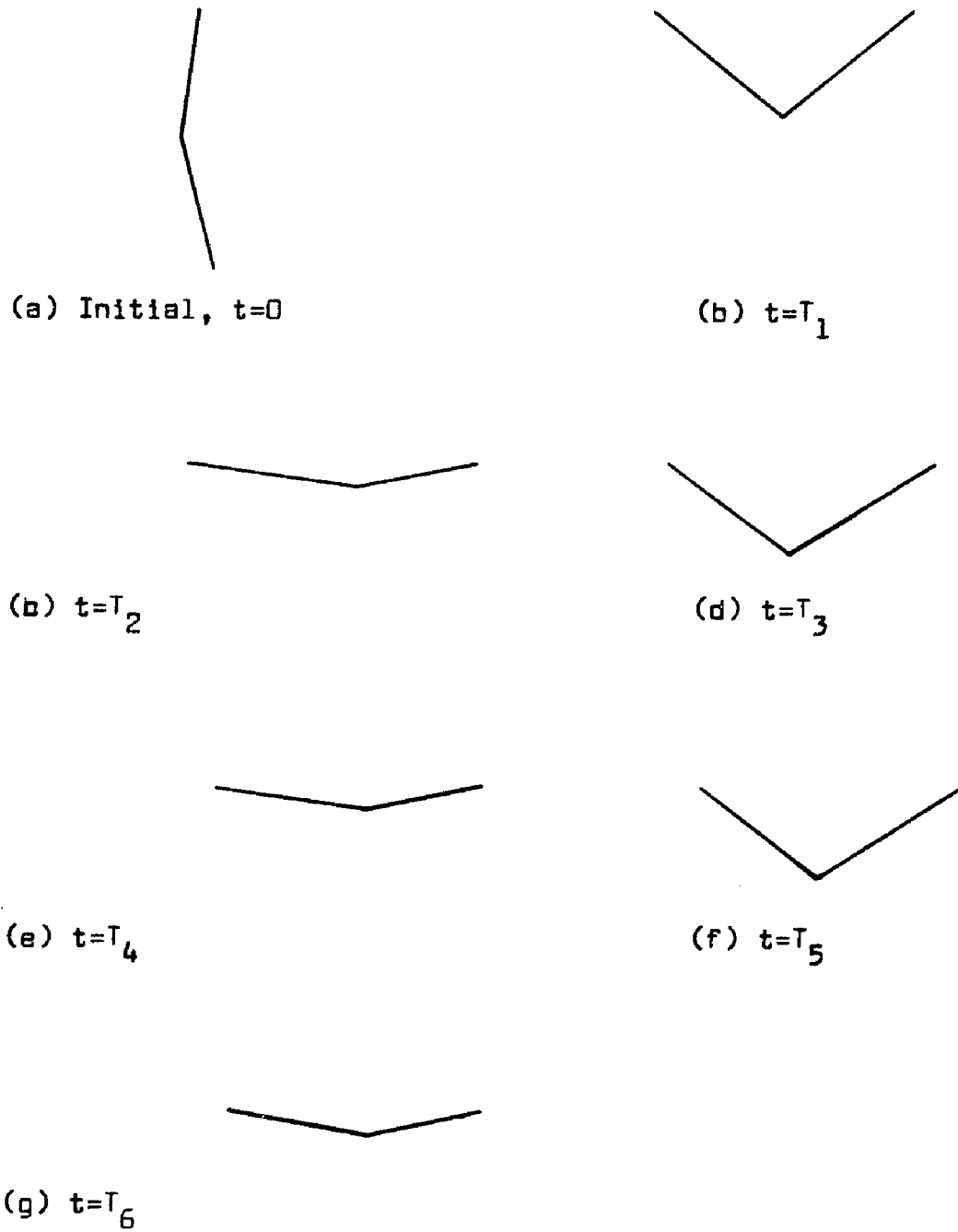


Figure 4.6 Seven Key Poses for "Knocking" Animation

## 5. ANIMATION BY USE OF MULTIPLE MAPPING OPERATORS

### 5.1 Achieving Character Movement

It has been shown that one piece of static artwork can be mapped onto an output plane by mapping functions that make the output image different from the input image, the input artwork. Animation can be produced by series of mapping functions whose influences are controlled by properly timed unit step functions. Next can be developed the method of producing animated figures by dividing the total image into sections and mapping each of the sections independently but in concert with the others.

As an example, let the total image plane be divided into six sections. A figure can be sectioned into six parts, the torso, the head, two arms and two legs. The six parts of the figure can be drawn on the six sections of the input plane with the maximum dimension of each aligned horizontally. For the purpose of this example place the near arm on the top section, the near leg on the second, the head on the third, torso on the fourth, far leg on the fifth and the far arm on the sixth section as in Figure 5.1. Each of the sections would need to be mapped by different functions in order to produce the desired output image which is the character in Figure 5.2.

A setup of certain parameters is necessary to position all of the sections properly so as to obtain the complete figure as the output image. The mapping function necessary to transform a typical section could contain the following operators. The parameter values of the magnifying operator would control the relative size of the particular member. The center of rotation is

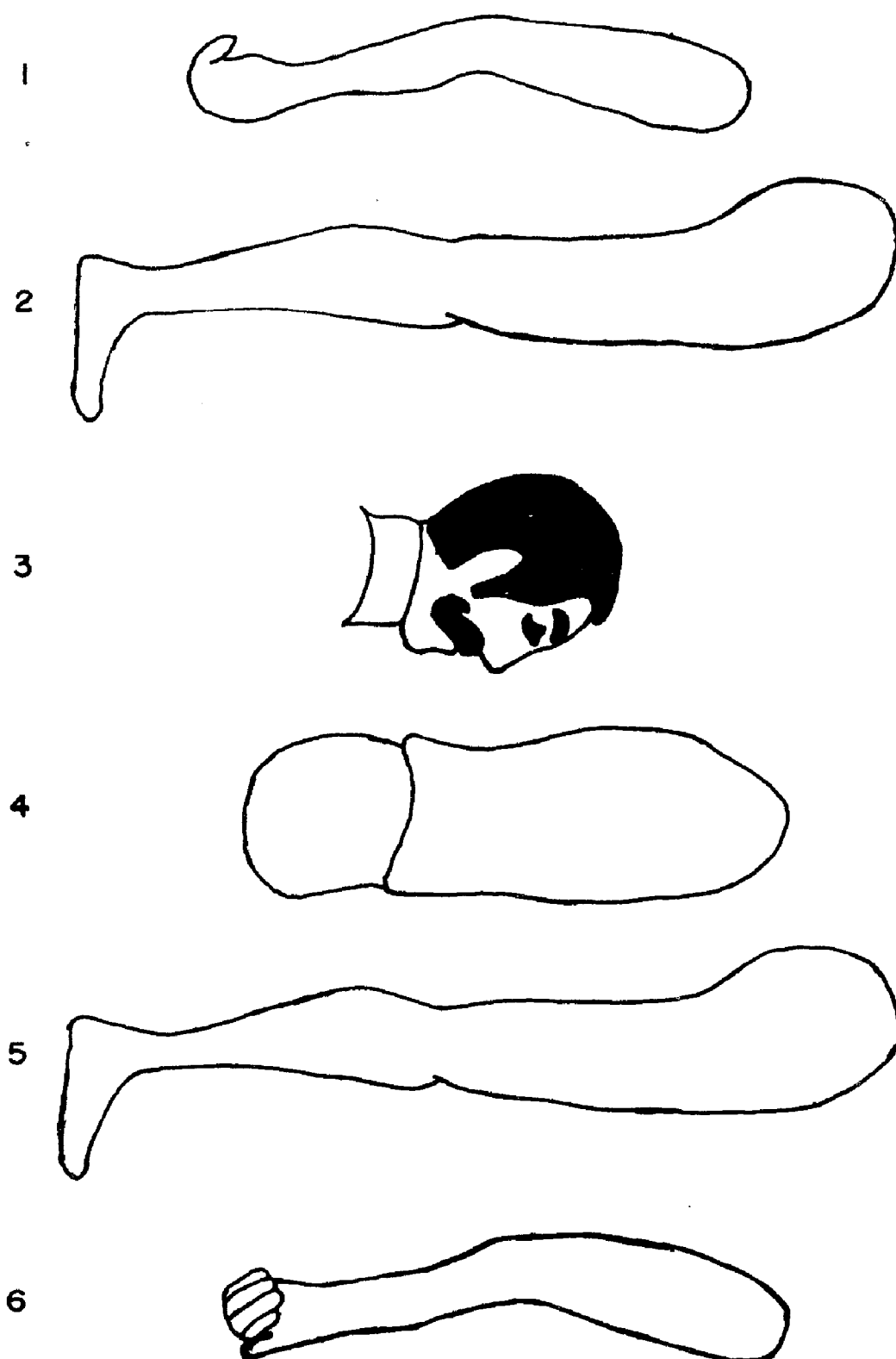


Figure 5.1 Input Image for Character

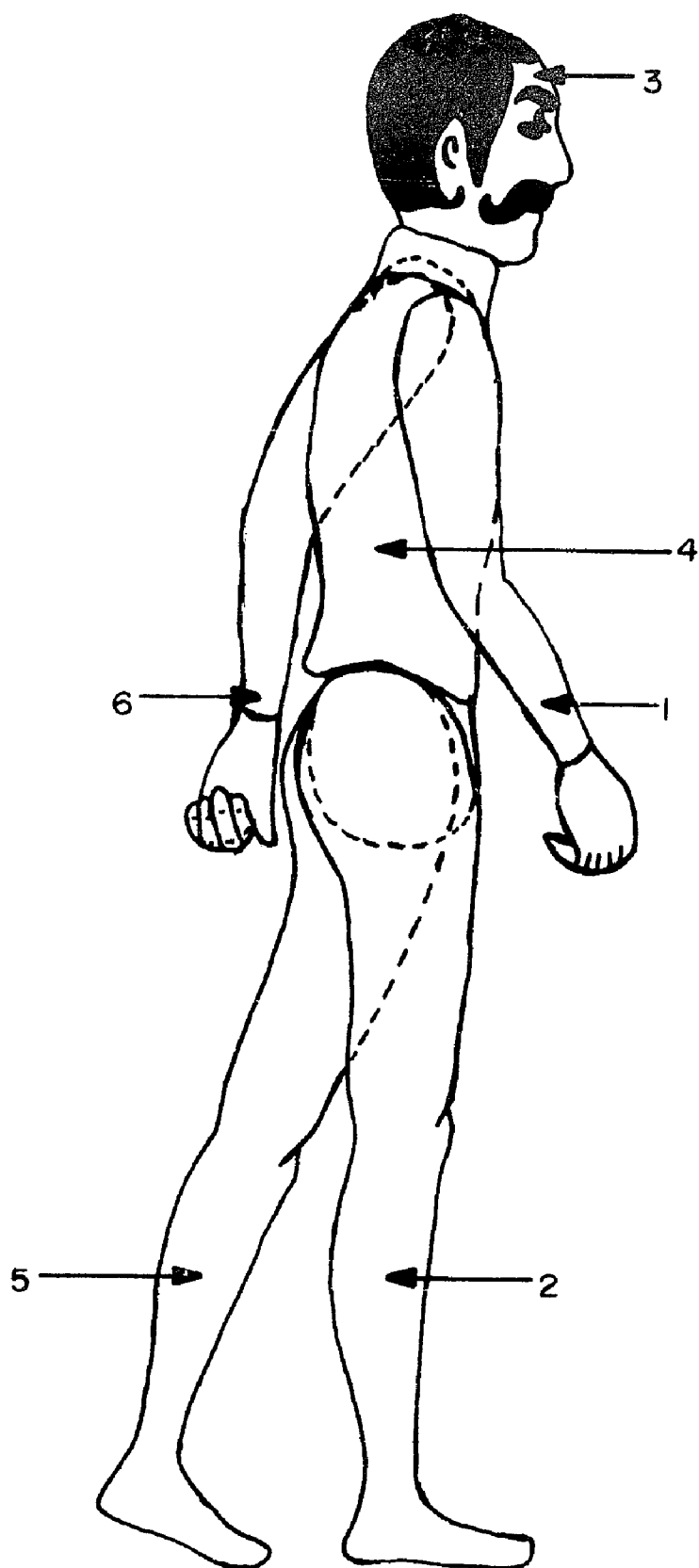


Figure 5.2 Output Image of Assembled Character

determined by the parameters of the initial translation operator. Next the angle that the member assumes is set by the parameter values of the rotation operator. Now the position of the member relative to the others can be controlled by the parameter values of another translation operator. A typical transformation equation for one section would be:

$$\bar{W} = \bar{T}[x_2, y_2] \bar{R}[\theta] \bar{T}[x_1, y_1] \bar{M}[a, b] \bar{Z} \quad (5.1)$$

Mapping the input plane onto the output plane with the mapping function just described will produce a motionless figure. Changing a parameter value in one of the mapping functions will change the appearance of the figure. Changing the horizontal size parameter of the magnifying operator for section one will shorten an arm. Changing the angle in the rotation operator for section two will rotate a leg. However, as long as the parameters are constants the output image will be that of a static figure with no animation.

If animation or motion is desired then the dependency of the arguments of the different mapping functions on time must also be specified. An equation describing a typical animation could be:

$$\bar{W} = \bar{T}[x(t), y(t)] \bar{M}[A(t), A(t)] \{ \bar{T}[x_{12}(t), y_{12}(t)] \bar{R}[\theta_1(t)] \bar{T}[x_{11}(t), y_{11}(t)] \bar{M}[a_1(t), b_1(t)] \bar{Z}_1 + \dots + \bar{T}[x_{52}(t), y_{52}(t)] \bar{R}[\theta_5(t)] \bar{T}[x_{51}(t), y_{51}(t)] \bar{M}[a_5(t), b_5(t)] \bar{Z}_5 \} \quad (5.2)$$

or the equation could be written as a summation:

$$\bar{W} = \sum_{\alpha=1}^n \bar{W}_n = \bar{T}_n[x(t), y(t)] \bar{M}[A(t), A(t)] \left\{ \sum_{\alpha=1}^n (\bar{T}[x_{\alpha 2}(t), y_{\alpha 2}(t)] \bar{R}[\theta_{\alpha}(t)] \bar{T}[x_{\alpha 1}(t), y_{\alpha 1}(t)] \bar{M}[a_{\alpha}(t), b_{\alpha}(t)] \bar{Z}_{\alpha}) \right\} \quad (5.3)$$



In this equation  $n$  is the total number of sections used for the figure.

Briefly, the equation for the output plane  $\bar{W}$  describes the mapping of the input plane  $\bar{Z}$ . It tells how it is possible to map a flat piece of artwork with the different movable parts of the desired figure placed on separate horizontal sections into a figure on the output plane. Actually the output consists of  $n$  flat planes arranged so that the images on the planes join together to form the figure. The individual sections are sized, offset, rotated and positioned and all  $n$  sections are sized and positioned by a common set of operators:

$$\bar{T}[X(t), Y(t)] \bar{M}[A(t), A(t)] \quad (5.4)$$

If shaping of the individual sections is required to add effect to the movement then the arguments of the various operators making up the total mapping function may be altered. Suppose the arm that is drawn on section one needs to be bent. Arguments of the magnifying operator and of the initial translation operator would need to be modified. The horizontal size argument would need to reflect the change in arm length during the sequence. The vertical displacement argument of the initial translation operator would need to have a section bending component whose amplitude is a function of the time of the sequence. For the example, the mapping that would produce the required arm bending would be described by the following equation:

$$\begin{aligned} \bar{W}_4 = & \bar{T}[x(t), y(t)] \bar{M}[A(t), A(t)] \bar{T}[x_{42}(t), y_{42}(t)] \bar{R}[\theta_4(t)] \\ & \bar{T}[x_{41}(t), y_{41}(t) + v(t, x)] \bar{M}[a_4(t) + s(t), b_4(t)] \bar{Z}_4 \end{aligned} \quad (5.5)$$

$V(t,x)$  is a function of both the time of the sequence and of the horizontal value of the output plane. Its purpose is to produce a bend in the output plane with an amplitude that varies with the time of the sequence.  $S(t)$  is a size factor that varies with the time of the sequence and it serves to vary the horizontal size, the length of the arm.

This technique of including time and/or position dependent factors in the arguments of the various mapping operators greatly increases the potential of this animation process. Without these factors the animated figures are merely stiff moving slabs similar to a figure constructed of cardboard with pins joining the sections at their rotation points. With these factors the members become much more lifelike because these factors cause the input plane to appear to be plastic when mapped onto the output plane. These factors can be included in the arguments of any of the mapping operators. The effect achieved depends on the operator used.

## 5.2 Fairing

In the animation technique discussed so far it has been assumed that all animation or movements are linearly dependent on time. That is, an arm rotating from one position to another has a constant rate throughout the time interval required to perform the rotation. Many times it is desired that a movement blends with its neighboring movements, no abrupt changes taking place. This property of letting the rate of the animation depend on the specific time of the sequence possesses several different names. Some call it "slow out, slow in" because the rate accelerates from zero at the start, achieves a maximum rate and then slows

back to zero at the end of the sequence. The term "fairing", an old craft word commonly used in the aircraft industry, is probably the most accurate available in the mathematical sense. (Halas,1959)

There are many possible relationships that can exist between the time of the sequence and the rate of animation. One that produces a very smooth fairing is described by the Versed Sine function.

Figure 5.3 demonstrates the relationship between a new variable  $g$  and  $t$ . The equation for the relationship is

$$t = 1 - \cos wg \quad (5.6)$$

Now if the variable  $g$  is used in the mapping functions, a linear progression in the value of  $g$  will translate into a non-linear progression of  $t$ . The new progression of  $t$  will effect small changes in the rate of change of the parameters as  $t$  starts from a value  $t = 0$ . The rate of change of the parameters will reach its peak at the midpoint of the sequence and decrease to a minimum value at the end of the sequence. Thus the effect of fairing has been achieved.

This is just an example of one of many possible fairing functions. It might be desired that the beginning of the sequence be faired to blend in with the previous sequence. One method of achieving this is to perform the calculations as if the sequence were really twice as long as it is. Then the rate of change of the parameters would be zero at the start of the sequence and would be maximum at the end of the sequence. Actually the number of different fairing functions is unlimited. All that is required is that the relationship between the linear passage of time and

the desired fairing be described. The relationship can be arbitrary and irregular.

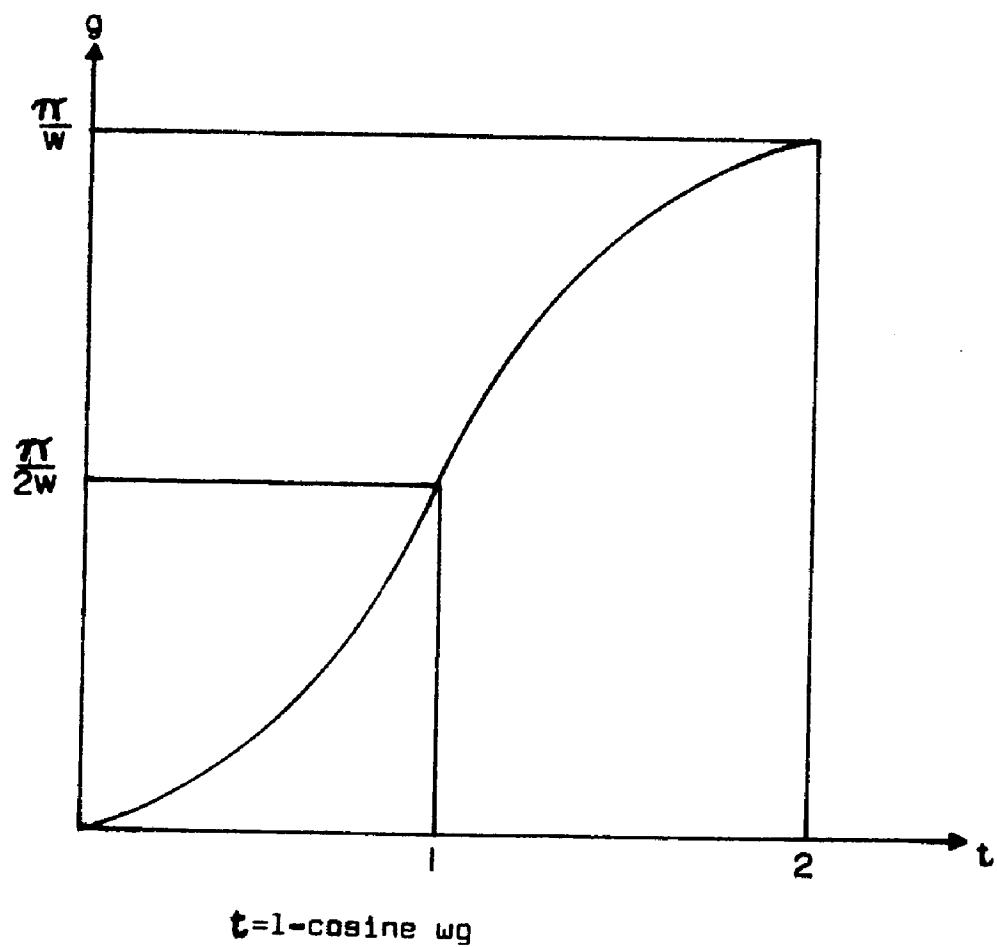


Figure 5.3 One Type of Fairing Function

## 6. IMPLEMENTING THE ANIMATION EQUATION

### 6.1 A Raster As An Electronic Plane

The ability to map a flat surface into an arbitrary orientation and shape is not common for a computer. Most digital computer CRT displays consist of series of dots that form lines and curves. Some also have the ability to draw arbitrary vectors which are straight lines that can have any length and direction. These can produce curves which consist of a series of short straight lines. Analog computer CRT displays usually consist of a continuous line that represents the solution of a differential equation. The traditional CRT displays of both digital and analog computers consists of straight and curved lines that can form both outlines of complex shapes and alphanumeric symbols.

A different kind of display system is necessary to produce planes and other complex surfaces. One electronic analog of a plane is a raster. A raster results when ramp voltages of different frequencies control the vertical and horizontal deflection of the electron beam of a CRT. Assume that the lower frequency ramp voltage controls the vertical deflection and a much higher frequency ramp controls the horizontal deflection. Then as the electron beam is deflected from the top to the bottom of the CRT by the vertical ramp voltage it is also deflected many times from the left to the right of the CRT by the horizontal ramp voltage. The display on the CRT then effectively consists of a number of straight parallel slanting lines. The number of these lines is determined by the multiple of the vertical ramp frequency that is the horizontal ramp frequency. The greater this multiple, the

more parallel slanting lines there are from the top to the bottom of the display. The more there are of these slanting horizontal lines the closer the raster display approximates a flat surface or plane. Conventional NTSC television systems have 525 of these horizontal lines in each frame and this produces the flat surface for the output images of television. A raster is used in a television camera to help convert visual information into a time-ordered electronic, luminance signal. It controls the movement of the electron beam across the target of the camera tube. Thus the target voltage, the luminance signal, is produced by the combination of the light intensity of the image focused on the target and the position of the electron beam which is controlled by the raster. The same raster is used in the monitor to control the position of the electron beam on the display tube. The time-ordered luminance signal controls the beam current and thereby the intensity of the light produced by the display tube. This combination produces an image on the monitor that is a duplicate of the image on the camera tube.

This system, with modification, makes it possible to produce animation by mapping. The input plane is the sheet of paper upon which the artist produces his artwork. It is scanned by a camera and then displayed on a monitor. If the raster in the monitor is the same as the raster in the camera then the displayed image is a copy of the input artwork. Therefore the output plane  $\bar{W}$  is equal to the input plane  $\bar{Z}$ .

Or

$$\bar{W} = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \bar{Z} \quad (6.1)$$

However, it is possible to make the displayed image look different than the input artwork by changing the characteristics of the raster used in the display monitor.

A plane that is produced by a raster can be described by a specific number of planes. By controlling the values of certain critical parameters such as the height and width of the raster, its angular position with respect to a reference and its horizontal and vertical positions, the entire raster is controlled. The concept of controlling the output raster or plane by controlling its parameters makes the amount of data necessary to specify each particular image in an animated sequence quite manageable. Each image is composed of the input artwork being displayed on the output plane. The output plane can be broken up into a finite number of separately controlled rasters so only a finite set of raster controls is necessary to describe each image.

## 6.2 Producing the Plane

It is possible to produce a raster with two resettable ramp generators. These could basically be integrators with a constant gain that are reset at regular intervals of time. One would be reset at the horizontal scan rate and the other would be reset at the vertical scan rate. The same reset pulses control the scanning signals of the input camera. If the input to the horizontal input of the display is the output of the horizontal ramp generator and if the vertical input of the display is the output of the vertical ramp generator and the video signal is supplied by the input camera then the displayed image is a copy of the input artwork. The block diagram of this operator is



shown in Figure 6.1

### 6.3 Magnification Operator

Now the size of the displayed image may be controlled both vertically and horizontally by controlling the relative size of the display raster. This can be implemented by inserting multipliers in the horizontal and vertical channels. The gain of the multiplier in the horizontal channel is  $a$  and the gain of the multiplier in the vertical channel is  $b$ . These gains can be controlled by a digital computer if the digital computer would supply input numbers to D/A converters. Then the outputs of the D/A converters would be the gain inputs to the two multipliers and would be equivalent to  $a$  and  $b$ . This operator is represented in Figure 6.2

### 6.4 Translation Operator

The translation operator is a relatively simple one to implement. Initially, the ramp voltages that produce the raster are summed with specific constant voltages in an analog summing amplifier so as to have the raster properly positioned. A translation operator merely moves the raster to some other specified location by summing in another different positioning voltage. If the positioning voltage is constant during the time required to display the particular section then it merely positions the section. If the positioning voltage is a function of either the vertical or horizontal position then it is an image manipulation signal.

The image on a particular section can be bent by translating the display raster with the translation argument being a function of the position relative to the raster. This

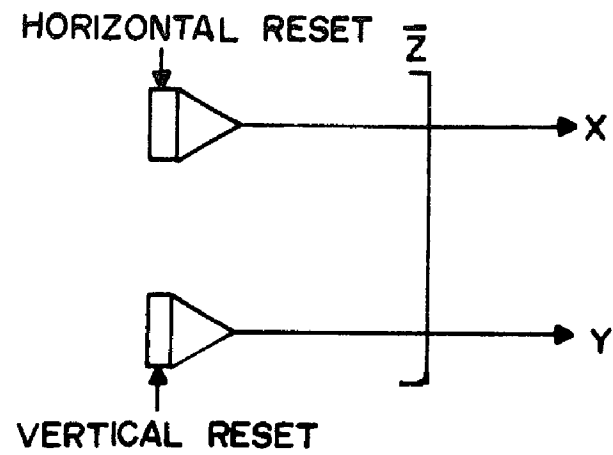


Figure 6.1 Block Diagram of a Raster Generator

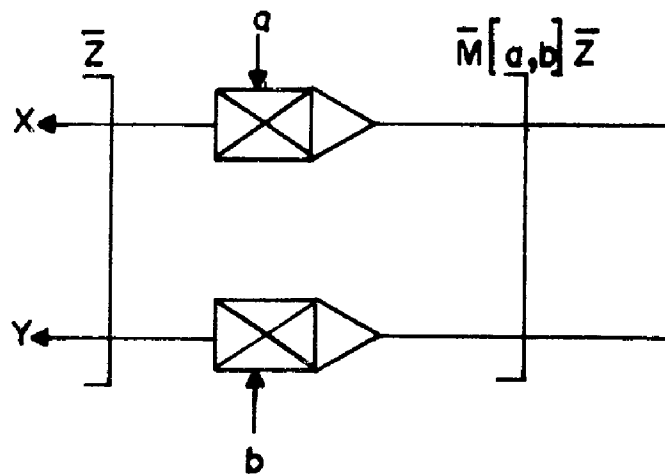


Figure 6.2 Block Diagram of a Magnifying Operator

function was discussed previously in Section 4.3. The triangular waveform needed to bend the arm can be generated by a function generator with certain characteristics. It must have the ability to be synchronized with either the vertical or horizontal raster generators. The type of output waveform must be selectable; sine wave, triangle wave, square wave or some other arbitrary wave shape. The phase of the waveform and its amplitude must be controllable. All of these waveform parameters may be controlled by a digital computer.

A signal having the just mentioned characteristics can produce desirable image manipulations when used in the argument, alone or with other components, of the translation operator. Bends can be gentle as produced by a sine wave or sharp as produced by a triangle wave. They can be positioned either by changing the frequency or the phase of the manipulation signal. Several different signals may be summed together to produce a more complex image manipulation. The signal could be a completely arbitrary one introduced by some means such as a light pen and then repeated in synchronism with the appropriate raster signal.

Therefore, it can be seen that the translation operator can position the raster and also can shape it. The same operator can do both by having two components in its argument as shown in Figure 6.3

## 6.5 Rotation Operator

The rotation operator is a more difficult one to implement. A minimum of four multipliers and an inverter and two summing amplifiers are required as well as a source to provide

the sine and the cosine of the desired angle of rotation. The necessary configuration is shown in Figure 6.4. The result of the rotation function operating on a plane can be examined.

$$R[\theta] \bar{Z} = \bar{W} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x \cos\theta - y \sin\theta \\ x \sin\theta + y \cos\theta \\ 1 \end{bmatrix} = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \quad (6.2)$$

Thus, it is determined that the horizontal component of the resultant plane  $\bar{W}$  is  $u$  which is equal to  $x \cos\theta - y \sin\theta$ . Similarly the vertical component  $v$  is equal to  $x \sin\theta + y \cos\theta$ . These results can be implemented in the following manner. The sum of the horizontal component of the initial plane,  $X$ , multiplied by the cosine of the angle of rotation,  $\theta$ , plus the inverted product of the vertical component of the initial plane,  $Y$ , multiplied by the sine of  $\theta$  is the horizontal component of the resultant plane. The vertical component of the output plane is the sum of  $X$  multiplied by the sine of  $\theta$  plus  $Y$  multiplied by the cosine of  $\theta$ . There are various methods of providing the sine and the cosine of the angle of rotation. A digital computer is capable of calculating these quantities and with the help of D/A convertors these quantities can be supplied as analog voltages.

## 6.6 Positioning Translation Operator

After the display plane has been rotated by the rotation operator it must be translated again so as to position it properly. The initial translation operator had as a constant component,  $X_1$  or  $Y_1$ , that determined the point about which the plane would rotate. Now the second translation operator is necessary so as to move this point of rotation to the proper

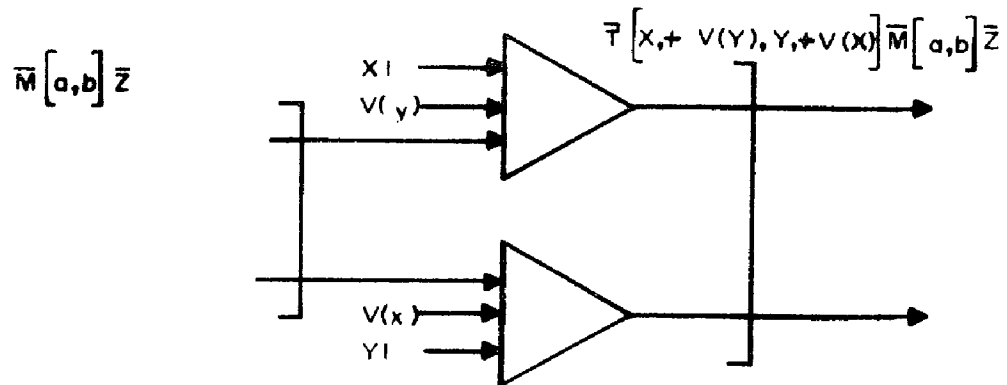


Figure 6.3 Block Diagram of a Translation Operator

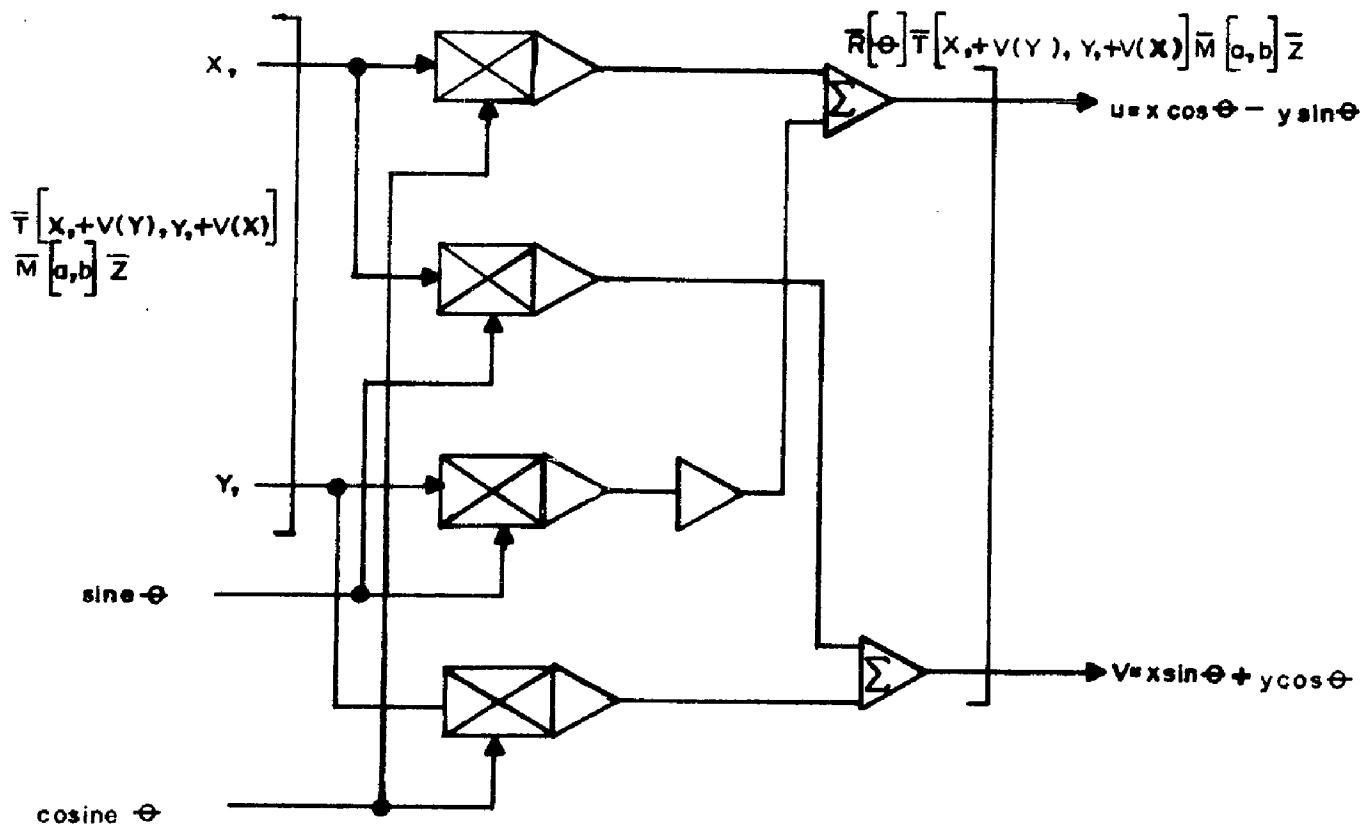


Figure 6.4 Block Diagram of a Rotation Operator

position. This can be accomplished with two summing amplifiers.

It has been shown that all of the necessary operations may be performed by a hybrid computer. The analog portions are controlled by the digital computer through the means of D/A converters and digitally controlled function generators. First the display plane, a raster, is generated and then it is manipulated by a series of mapping operators. These operators enable the precise shaping of the output image whose detail is supplied by the input artwork scanning camera.

#### 6.7 Determining Operator Arguments

So far in this development only the mapping of a single plan or raster has been considered. However, it was previously shown that a character could be animated by placing selected portions of the character on different rasters and then controlling the parameters of these rasters in accord with each other. To accomplish this, each of the sections would need to be mapped by different functions. This is not a problem as the rasters can be drawn sequentially in time and arbitrarily in space. Each frame, or picture, and therefore each raster, must be drawn 30 times per second assuming a television output.

Recall that the equation describing a typical animation was:

$$\begin{aligned} \bar{W} &= \sum_{\alpha=1}^n \bar{W}_\alpha = \bar{T}[x(t), y(t)] \bar{M}[A(t), a(t)] \\ &\quad \left\{ \sum_{\alpha=1}^n \bar{T}[x_{\alpha 2}(t), y_{\alpha 2}(t)] \bar{R}[\theta_\alpha(t)] \right. \\ &\quad \left. \bar{T}[x_{\alpha 1}(t), y_{\alpha 1}(t)] \bar{M}[a_\alpha(t), b_\alpha(t)] \bar{Z}_\alpha \right\} \end{aligned} \quad (6.3)$$

This was the correct equation for the idealized system where the

individual sections were rubber sheets. A physically realizable equation would allow for the practical constraints such as the fact that the effect of motion is produced by 30 different pictures per second drawn in the television format of two interlaced fields for each individual frame or picture.

However, the sequential nature of a television picture is an asset. When section one is being mapped into the final image the transformation equation needs to contain section one's parameters. After the time necessary to draw section one is over then the parameter values can change from those of the first section to those of the second. After section two is drawn then the parameter values can change from those of the second section to those of the third section. This process continues until all sections have been mapped into the final image. This cycle repeats for each complete field.

A step function can be employed to indicate this operation. First define:

$$T = t \bmod T_m = t - (T_m \lfloor t/T_m \rfloor) \quad (6.4)$$

where  $T_m$  is the period of one frame

Then the step function

$$h(T) = h(T - T_1) \quad (6.5)$$

defines the time interval for the drawing of the first section

$$h(T - T_1) = h(T - T_2) \quad (6.6)$$

defines the period of the second section and so on. If a total of five sections are used then the period of the fifth section

is defined by

$$h(T-T_4) - h(T-T_5) \quad (6.7)$$

where  $T_5 = T_m$ .  $T_m$  is approximately equal to 16.67 milliseconds for a 60 field per second rate.

Now the equation for a nonanimated picture of the character contained on five sections can be written as:

$$\begin{aligned} \bar{W} = \bar{T}[x, y] \bar{M}[A, A] \sum_{\alpha=1}^n [h(T-T_{\alpha-1}) - h(T-T_{\alpha})] \\ \bar{T}[x_{\alpha 2}, y_{\alpha 2}] \bar{R}[\theta_{\alpha}] \bar{T}[x_{\alpha 1}, y_{\alpha 1}] \bar{M}[a_{\alpha}, b_{\alpha}] \bar{Z}_{\alpha} \end{aligned} \quad (6.8)$$

This equation describes a static picture but the only modification necessary to introduce movement is to make the mapping function parameters vary as time increases. These parameters would not be continuous functions of time but would attain specific values for each individual frame of the scene. The equation for a five second scene of animation can be written now as:

$$\begin{aligned} \bar{W} = \sum_{m=1}^{150} \bar{T}[x(t_m), y(t_m)] \bar{M}[A(t_m), A(t_m)] \\ \left\{ \sum_{\alpha=1}^5 [h(T) - h(T-T_{\alpha})] \bar{T}[x_{\alpha 2}(t_m), y_{\alpha 2}(t_m)] \bar{R}[\theta_{\alpha}(t_m)] \right. \\ \left. \bar{T}[x_{\alpha 1}(t_m), y_{\alpha 1}(t_m)] \bar{M}[a_{\alpha}(t_m), b_{\alpha}(t_m)] \bar{Z}_{\alpha} \right\} \end{aligned} \quad (6.9)$$

where  $t_m$  is the time at the start of each frame.

The equation for the 150 frames comprising the five second scene indicates that each of the mapping functions parameters are determined at the beginning of each frame. Two of the operators affect the entire frame and so the arguments of the final translation and magnifying operators remain constant



during the entire frame. All of the other operator arguments, however, are changed for each individual section. Thus, each section can be positioned and shaped differently for each frame and each total frame can be sized and positioned arbitrarily.

There is no great difficulty in implementing this last equation with the hybrid computer. All that is necessary is that the voltages representing the various parameters change for each section and each frame. This is possible either with an analog or a digital multiplexer. Then the voltage representing the "a" parameter of the Magnifying operator would have a particular value equal to  $a_1(t_m)$  during the time that the first section is drawn, equal to  $a_2(t_m)$  during the time for the second section, equal to  $a_3(t_m)$  during the time for the third section and so on.

If there is no movement then all of the  $a_1(t_m)$ 's are equal as  $m$  varies from 1 to 150, unchanging for each frame. However, when there is movement, each of the 150  $a_1(t_m)$ 's may be different.

7.1 Requirements of Animation

The capability to make static images appear to be characters that can move has been discussed. But the mere movement of characters is not animation. Animation is really a language that enables an animator to tell a story to his audience. The animator creates a series of images, each different from the other, and by changing these images rapidly he creates the illusion of movement. The type of movement that the audience sees is controlled by the types of images that are used and the timing with which they are displayed. These two factors determine the characteristics of the movement.

There are several types of movement that can be depicted by the use of animation. One is simulation where the movement must be realistic in order to be effective. This means that everything must obey the laws of nature. The forces of gravity and friction, the effect of momentum and the relationship between physical size and speed of locomotion must be considered. One method of obtaining realistic simulation of a humanoid is to use an anthropometric harness that measures particular parameters of movement of a person. These parameters, if properly chosen, can then be used to control the movement of an animated character. Since the person in the harness is subject to the laws of nature, the animated character's movements will also seem to obey these restrictions. Another way to get realistic motion is to have a controller that has the laws of nature programmed into its operation to control the movement. One example of this is to

have the voltage output of an analog computer control the vertical movement of an image of a ball. The ball, or image of the ball, can then appear to be dropped from a height. By properly adjusting the potentiometers that determine the elasticity of the ball and the force of gravity in the differential equation describing the motion of a ball, the ball can appear to be reacting to the laws of nature in an accurate manner. Digital computers can also be programmed to produce outputs that have taken the physical restrictions of motion into consideration. These outputs can be converted into the proper form and then used to control the parameters that cause the image to appear to move realistically.

Another type of movement that can be depicted with animation is that of a cartoon character. This movement can be at the same time similar to that of simulation and also quite different. One reason for the difference is that a cartoon character does not necessarily exist in a world governed by the same physical laws as ours. A cartoon character can be crushed as flat as a piece of paper in one scene and be completely normal in the next scene. He can change directions of motion in mid-air, he can leap over buildings, can stroll and live underwater or out in space, he can do all manner of things that mortals cannot.

The movement of a cartoon character can be quite similar to that of a simulated figure in that most of the time it can behave much as a mortal does. However, even when the movement is quite similar to that of real objects or creatures it can be quite different. The movement of a cartoon character usually has some exaggerated characteristics included. A Donald Duck hammer-

ing a nail will spend most of his time winding up for the blow and then almost instantaneously strike the nail with his hammer. This feat violates the physical law of momentum as much more force than a small duck could generate would be necessary to change the direction of the movement of the hammer and get it to move the distance to the nail head in such a short period of time. However this action can be quite consistent with the world that Donald Duck inhabits. Essentially, cartoon character movement is a caricature of movement in the real world.

The important factor in cartoon character animation is not exactly what the images are that comprise the animation but rather what the viewer thinks it is that he sees. A successful cartoon produces an illusion in the minds of its viewers that tells the story. Good animation creates this illusion without distracting the viewer with the methods or techniques used. For example a running character can be produced with several different types of images. One technique is to scoot the character smoothly past the background with three or four legs, equally spaced, rotating rapidly to create a blur of moving legs. Here the only movement consists of the spinning legs and the translation of the entire body past the background. Another method is to have three or four views of the legs in their different poses during the running action. Once again the character is moved past the background only this time the body can bend and bob as the different views of the legs are used in their proper sequence. This technique yields a slightly more sophisticated image of a running character.

Both of these methods require only a single drawing of the body. The first method also requires only one drawing of the legs that can be rotated so it can relatively easily produce animation. The second method requires three or four drawings of the legs but since they can each be used several times the difficulty of animation is simplified. An animator's skill is required to create the desired illusion of a running character. His special knowledge and experience enable him to choose the proper leg positions to give the desired effect.

One other technique is to move all portions of the characters body just as they would move in real life. Then the arms and legs bend, stretch and rotate and the body can bob during the course of a stride. This is at once the simplest and the most complex technique. It is the simplest since the illusion of a running character is created by mimicking the movements of a live character. However, it is the most complex because each of the views is a different image and requires a completely different drawing if done by conventional animation.

## 7.2 Compound Programming

Creating the different images with a hybrid computer requires complex programming. A leg when moving from one position to another must bend, change horizontal size and rotate. At the same time the other leg may be straightening, stretching and rotating. The arms require the same type of movements and the body also may rotate slightly, change horizontal size and vertical position. All of these movements must occur in the proper time sequence in order to produce the desired image. Compound program-

ming means that the various sections must be animated by action produced by several different operators. The amount of each operator's influence and the timing of its occurrence must be carefully programmed. Adding to the complexity is the need for the various sections to move in synchronism, each being animated by the action of several different operators.

This is a technique that minimizes the number of decisions that an animator need make while producing animation. In compound programming only one key parameter need be specified for the animation to be defined. This decreases the number of decisions that an operator needs to make while programming a sequence which means that it takes less time.

As an example once again take an arm that is to bend. The artwork for the arm is orientated horizontally on the section. A bending motion can be achieved by a combination of a bending translation and a decrease in horizontal size. This process was previously described in Section 4.4. However, this technique is successful only for bends of up to about 45 degrees. If the bend exceeds 45 degrees then the arm becomes too distorted to maintain the illusion that it is a bending arm. The reason for the loss of illusion could be because the mapping functions producing the bend result in a non-conformal mapping. The prominent characteristic here is the fact that angles are not preserved by the mapping process. This fact destroys the illusion that the animated arm is real. The mapping produced by the bending translation maintains point A directly above point A' and point B directly above point B' as shown in Figures 7.1 and 7.2. If the

horizontal magnifying operator of the display raster was a function of the vertical position and the amount of bend then a better illusion would result. The effect achieved by the magnifying operator alone is shown in Figure 7.3 and the combined effect is shown in Figure 7.4. The result is a much better approximation of the effect of a conformal mapping process since angles are preserved much better. The image retains the illusion of a bending arm through a much larger bending angle.

The best results will be obtained if the peak amplitude of the bending translation, the amount of raster shaping and the amount that the horizontal size is decreased are all controlled by one bending parameter. Another factor that must be controlled to maintain image integrity during a deep bend is to make the peak amplitude of the bending translation also dependent on the amount of raster shaping. The effect of this last factor is demonstrated in Figures 7.5 and 7.6.

Therefore in order to obtain an effective bend three different decisions must be made. The peak amplitude of the bending translation, the amount that the horizontal size is decreased and the amount of raster shaping. In addition, the peak amplitude of the bending translation must be modified by a factor dependent on the amount of raster shaping. These decisions if made in the usual manner will take a certain amount of time. If much use is going to be made of the bending mapping then some form of simple compound programming could be utilized. It could be accomplished by either hardware or software. In both cases after the bending routine is realized by compound programming

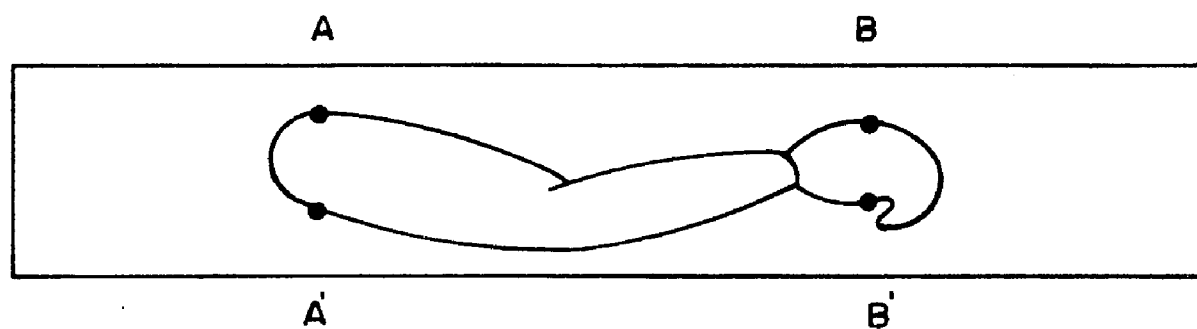


Figure 7.1 Artwork for Arm Orientated Horizontally on Section

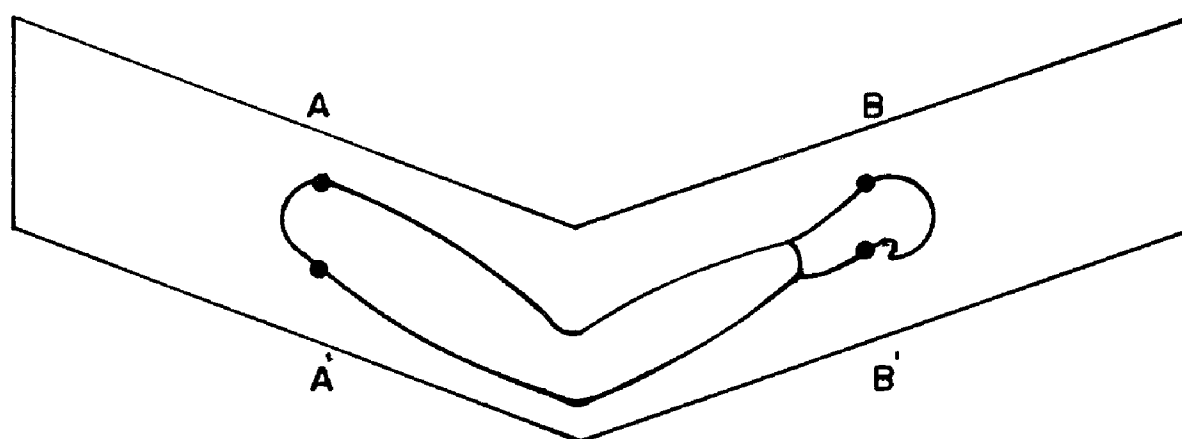


Figure 7.2 Arm Bent Using Only Vertical Bending Operator



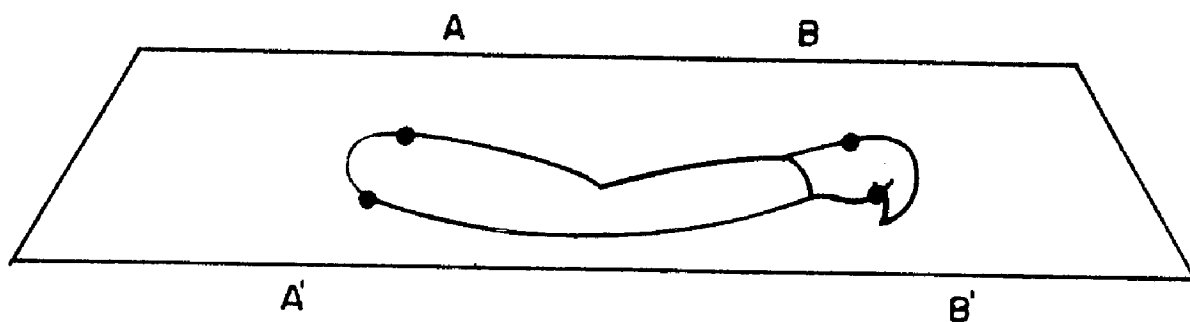


Figure 7.3 Arm Distorted by Raster Shaping

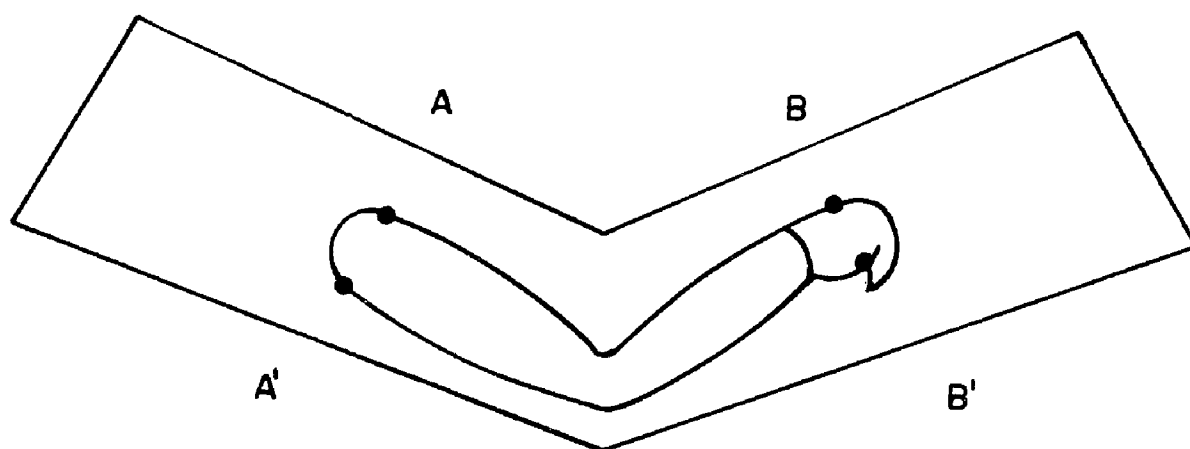


Figure 7.4 Arm Bent by Raster Shaping and Vertical Bending Operator

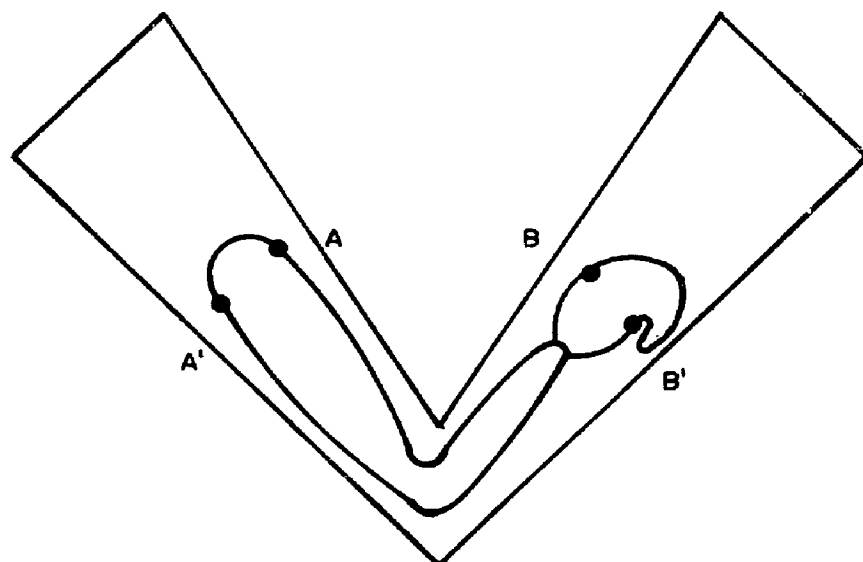


Figure 7.5 Arm Bend With a Fixed Amplitude Bending Translation

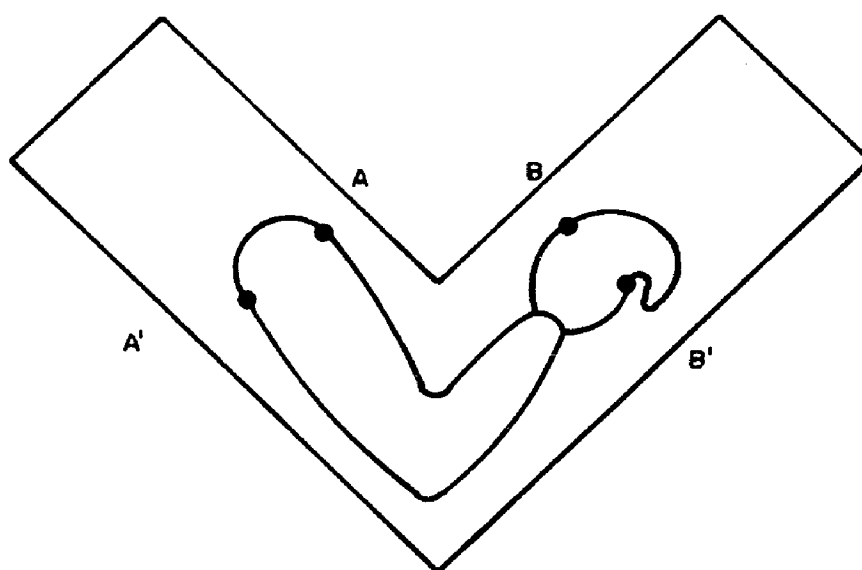
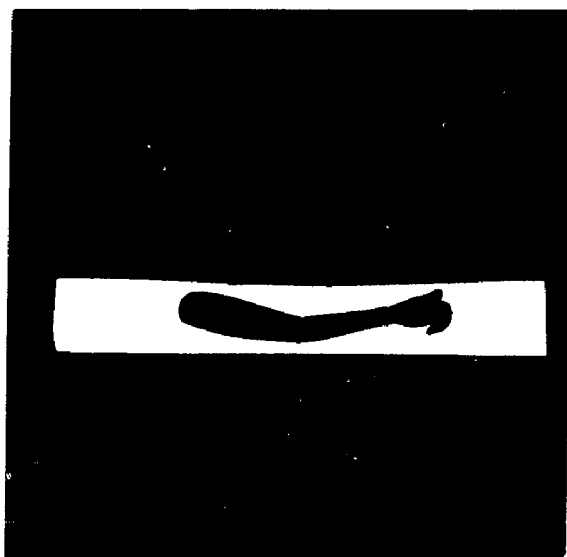


Figure 7.6 Arm Bend With a Dependent Amplitude Bending Translation

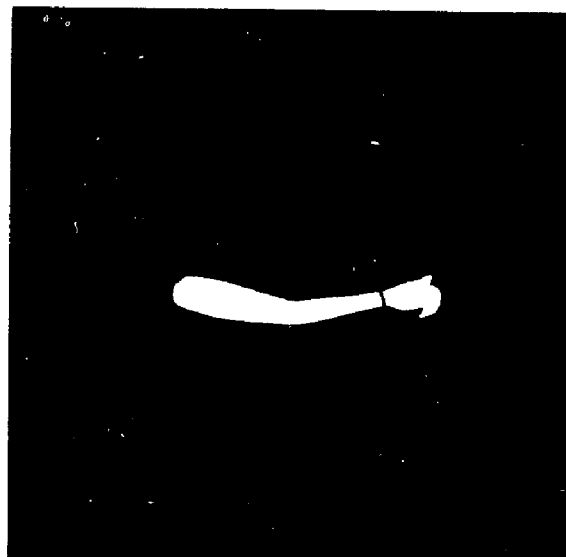
then only the one control that determines the amount of the bend need be adjusted. This means that only one decision need be made by the animator and that decision will affect all of the pertinent operator arguments so that the result is a maximally realistic bend. Examples of bends achieved with and without the compound bend operator are shown in Figures 7.7 through 7.12.

Even a simple compound program such as the one for bending has two advantageous attributes. First, the speed with which an animation sequence can be programmed is increased because fewer decisions and control adjustments need to be made. Second, the animator does not need to be as knowledgeable about how the desired effect is achieved. The animator does not need to know that the arguments of three different operators should be controlled. He only has to be concerned about whether or not a bend is desired. If it is, then how much of a bend is needed is his only decision.

Another way of stating it is that without compound programming the animator must control the computer with machine language. Compound programming enables the use of a more sophisticated language. This makes the computer much easier to use as was demonstrated by such sophisticated digital computer programming languages such as Fortran, Algol and Cobol. An analogy exists between the programming that enables compound control and the compiler program of a digital computer. Both convert the instructions used to control the computer into machine language to which the computer can respond.

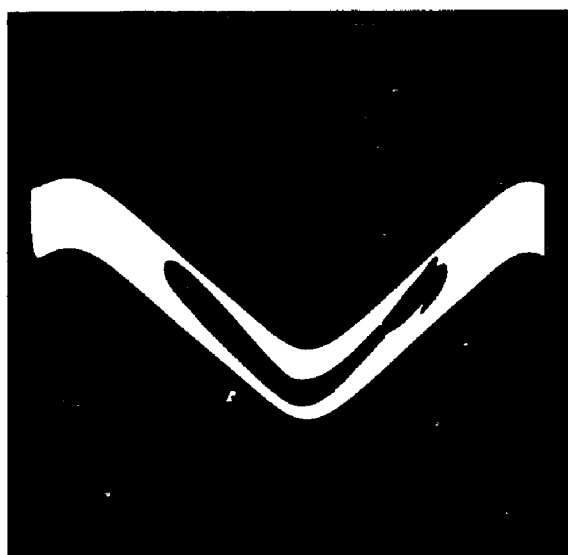


(a)

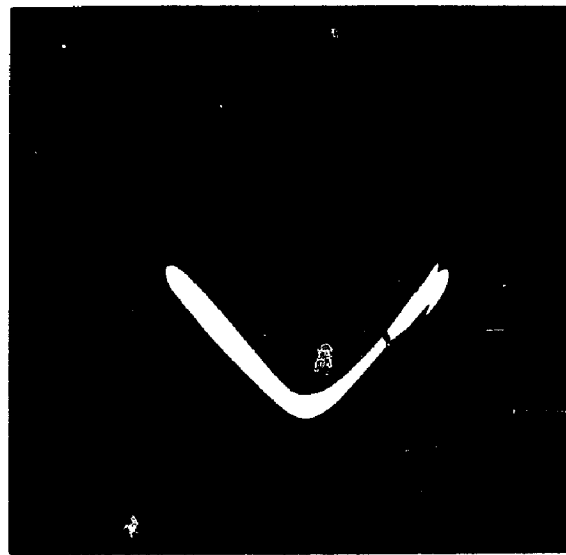


(b)

Figure 7.7 Artwork of Arm With no Bend  
(a) Negative of Artwork on Raster  
(b) Positive of Artwork



(a)



(b)

Figure 7.8 90 Degree Bend With Vertical Translation Operator  
Only  
(a) Negative  
(b) Positive

### 7.3 Methods of Implementing Compound Operators

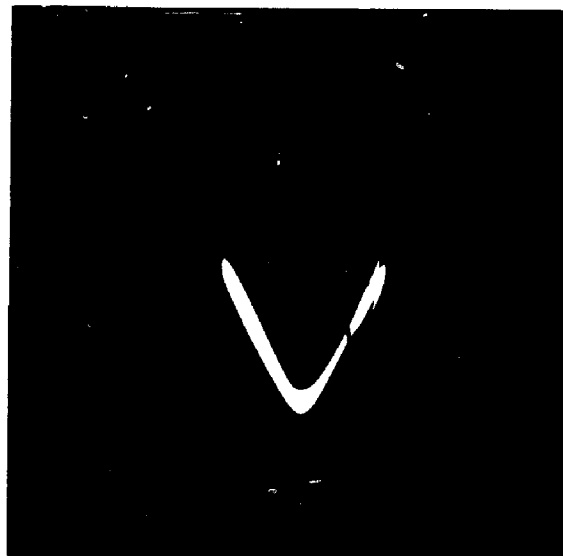
As was stated, compound programming can be achieved either with software or with hardware. The software approach increases the difficulty in building the system program. The digital computer must transform the one control variation into a number of operator argument variations. For example, when the bending control is activated the computer must know that three arguments are to be controlled by the one bending parameter. If the bend is increased then the peak amplitude of the bending translation must be increased, the amount of raster shaping must be increased and the horizontal size must be decreased. All of these instructions must have been pre-programmed into the system operating program. This programming uses memory locations in the computer so the size of the digital portion of the hybrid computer may affect the decision as to the method of implementing the compound approach.

A hardware approach to compound programming can also be used. The one parameter control may be converted to a voltage with a D/A converter. This voltage can be amplified, attenuated or inverted or different combinations of these operations may be necessary. The resultant voltage can then be channeled to affect the horizontal size, the raster shaping and the bending translation's peak amplitude. This approach decreases the load on the digital section as only one parametric control need be recognized and remembered.

However, once the system has been programmed by the hardware approach it is very difficult to change it. A program



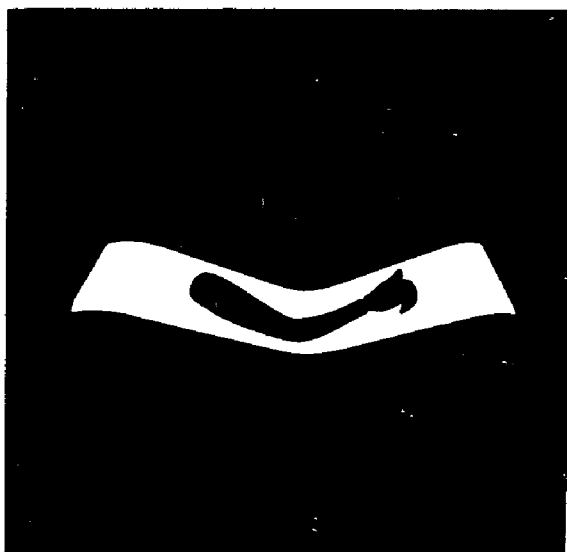
(a)



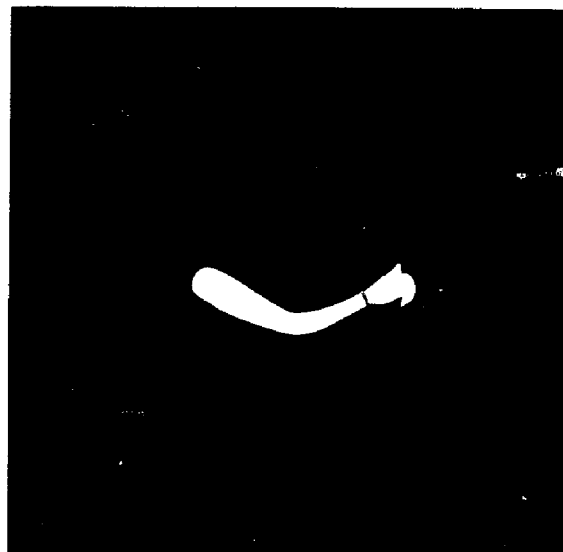
(b)

Figure 7.9 135 Degree Bend With Vertical Translation Operator Only

(a) Negative  
(b) Positive



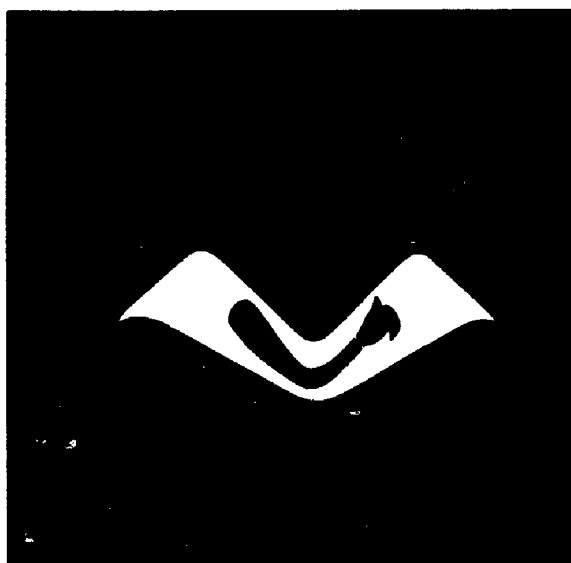
(a)



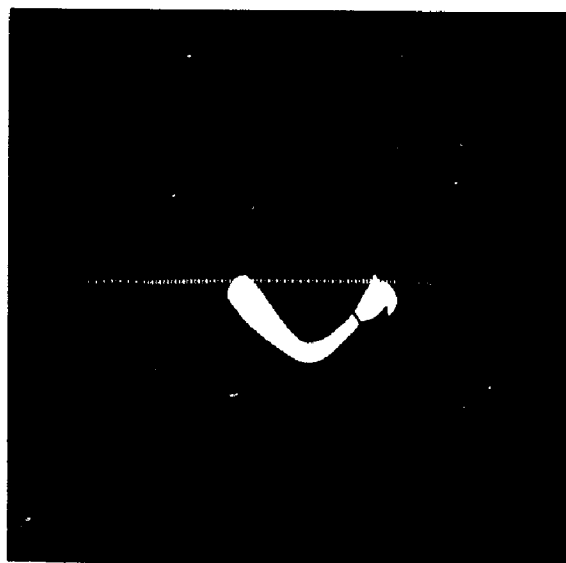
(b)

Figure 7.10 30 Degree Bend With Compound Bend Operator

(a) Negative  
(b) Positive

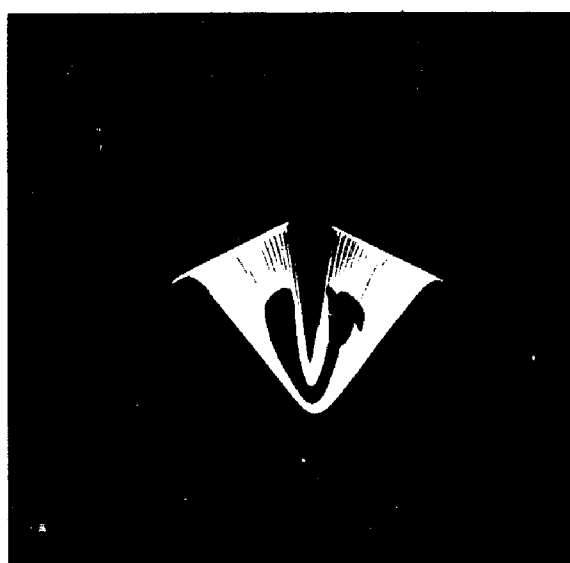


(a)

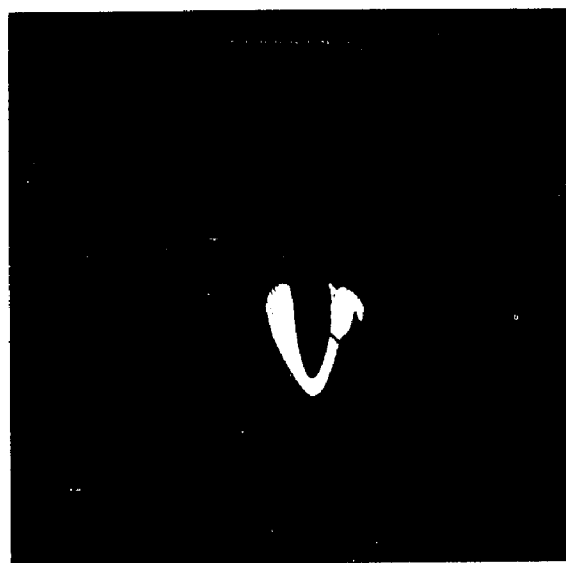


(b)

Figure 7.11 90 Degree Bend With Compound Bend Operator  
(a) Negative  
(b) Positive



(a)



(b)

Figure 7.12 150 Degree Bend With Compound Bend Operator  
(a) Negative  
(b) Positive

change requires a change in the circuit configuration. It may mean changing components to vary the gain or the attenuation. Amplifiers may need to be added or to be eliminated. In either case the circuit must be altered. This is difficult to do in a computer constructed with modern technology. The circuits are usually contained on printed circuit boards so that changing the configuration results in a lengthy, costly process of re-designing the p.c. boards. The interconnections between the p.c. boards may also need to be changed.

Some of the factors that must be considered then are the probability of a program change and the number of computers that may need to be changed. A change in the system program can be developed on one machine and then distributed to all other computers. Hardware changes could be developed on one machine but would have to be performed on all machines. If only one machine is involved and temporary circuit modifications are acceptable, then the hardware approach is quite satisfactory.

Compound programming can help to maintain other illusions of realistic movement as well as that of bending. It is possible to create the illusion that a section is rotating around the horizontal axis by causing the vertical size to decrease to zero and then to increase in the negative direction to the same absolute amplitude. This is effective if the vertical dimensions are small enough so that the absence of perspective is not disturbing. Perspective can be added with the use of compound programming. The horizontal size must be dependent on the relative vertical position in order to create a good



illusion of rotation about the horizontal axis. Similarly, the vertical size should be dependent on the relative horizontal position in order to maintain a good illusion of rotation about the vertical axis. In both cases the illusion is of a flat, two dimensional sheet but the sheet seems to rotate about horizontal or vertical axis instead of merely having size changes. These effects are shown in Figures 7.13 and 7.14.

This type of rotation could be produced by compound programming. A switch could be positioned to produce horizontal or vertical rotation and then a single control could adjust the amount of perspective. The amount of perspective would be dependent on the distance between the viewer and the object and on the size of the object.

Large bends and rotation of large planes about horizontal or vertical axis are two illusions that can be maintained by the use of compound programming. As was previously stated compound programming in a hybrid computer is analogous to the development of a language for a digital computer. It can also be instrumented quite effectively by hardware.

#### 7.4 Hierarchical Programming

There is another tool that is available and that can increase the animators efficiency. This is the technique of hierarchical programming and in effect it expands the concept of compound programming. Hierarchical programming essentially involves animation subroutines. It is possible to use pre-determined operator argument values to obtain a nominal animation cycle if certain conditions which will be discussed are met.

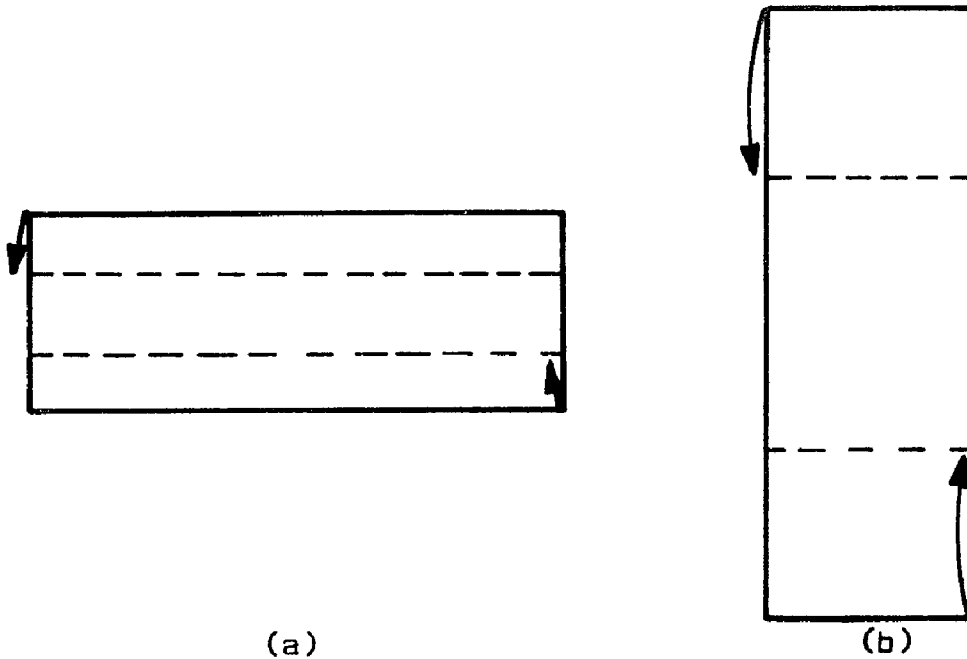


Figure 7.13 Effect of Rotation About Horizontal Axis by Varying Vertical Size  
 (a) Acceptable Illusion for Small Vertical Size  
 (b) Poor Illusion for Large Vertical Size

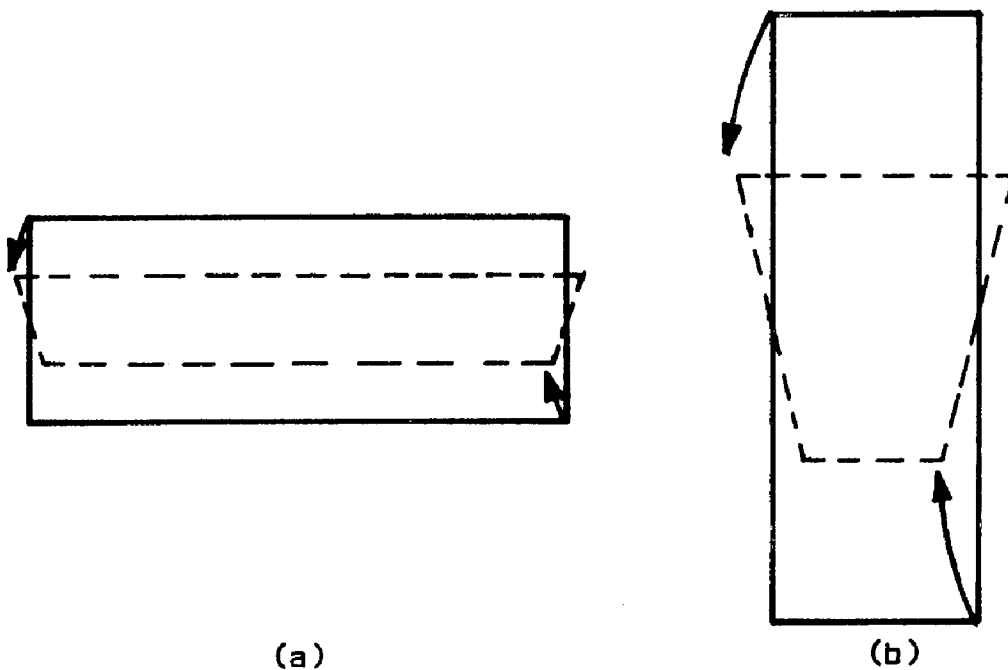


Figure 7.14 Effect of Rotation with Compound Operator  
 (a) Good Illusion for Small Vertical Size  
 (b) Good Illusion for Large Vertical Size

When the subroutine is activated then specific parameter values must be set. The result will be one cycle of animation that can then be modified by changing arbitrary operator argument values.

Hierarchical programming may best be explained by examining one specific example. Suppose it is desired to make the formation of a walk cycle quicker and easier because of its frequent use. The first step would be to set up certain standards for the input artwork.

The artwork for the character will need to be sectioned as it is in Figure 5.1. The artwork will need to be positioned carefully on the cell. The relative placement of the different members of the figure will need to be consistent. If the master cell were so set up then the right arm must always be on section one, the right leg always on section two, the head on section three and so on. The size of each section must be fixed and the absolute length of each member must remain consistent from one figure to another. Some reference point on the cell must be established. The animation peg holes on a standard animation cell serve this purpose. Another set of reference points that must be set for each member is the axis of rotation. The axis point must remain fixed and the artwork for each member drawn about that fixed point. These fixed parameters, axis or center of rotation and section lengths must always have the same position with respect to the reference point. Any kind of figure may be used as long as the artwork of the members is placed properly on the cell. It could be a Viking, a policeman, an Indian or a modern day businessman as long as the artwork con-

forms to the restrictions mentioned.

Now suppose this figure is to be animated so as to create the illusion that the figure is walking. In the first frame the different members must be positioned relative to each other so that the figure is assembled. This can be accomplished by adjusting the arguments of the various operators. Initially the value of the arguments could be derived by varying them until the figure assumed the desired image. Then a walk cycle could be programmed by normal operation.

There are eight key positions in one stride of a nominal cartoon walk cycle as shown in Figure 7.15. Here a key position is defined as one where there is a change in the sign of the rate of an operator's argument. One example is when an arm changes direction of rotation. Another example is when a head reaches the upper or lower limit of its vertical movement.

One technique of producing animation with a mapping system is to have the artist/operator construct the key frames, frames that portray the figure in its key positions. Then the computer can distribute the difference between argument values at the key frames among the frames between the key frames. The number of frames between key frames can be specified by the operator. The operator can also specify the manner in which these differences are distributed. There can be a linear relationship between frame number and argument value or the relationship can be arbitrary. This feature is the one that determines the type of "fairing" used to achieve the desired movement.

It is assumed that this technique has been implemented

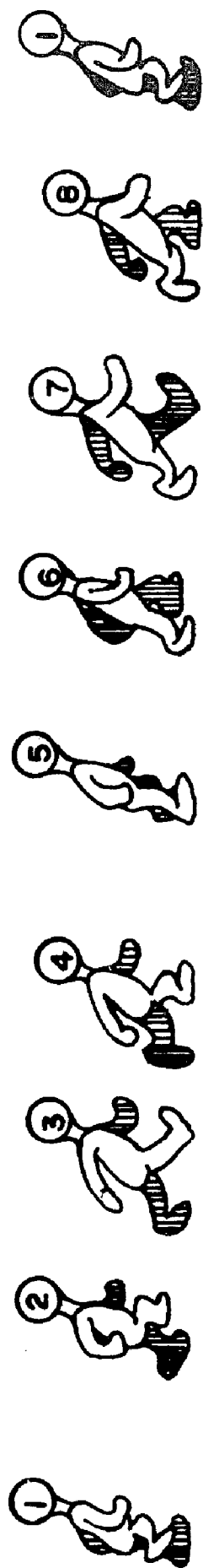


Figure 7.15 Eight Key Frames in a Typical Cartoon Character Walk Cycle

by either or both hardware and software. Appendix A describes such a system. It is called CAESAR.

The first step in producing a walk cycle is to assemble the various components of the character into an image that is a close approximation to the first key frame of the cycle. This set of operator argument values are the arguments that define the initial frame of an animation sequence. Next the operator arguments are adjusted so that the character assumes the configuration of the second key frame and is the final frame of the animation sequence. The speed of the walk is controlled by the number of frames between the initial and final frames of the animation sequence. The operator arguments for these intermediate frames, called inbetweens by animators, are calculated by the computer. After the computer has calculated and stored these frames then a portion of the walk cycle has been created.

Now another animation sequence will derive the operator arguments necessary to produce the animation between the second and third key frames. The second key frame was the final frame of the first animation sequence and the initial frame of the second animation sequence. In like manner the third key frame is the final frame for the second animation sequence and the initial frame for the third animation sequence. This same procedure continues until the eighth animation sequence. Here the eighth key frame is the initial frame and the first key frame is the final frame. This animation sequence completes the cycle as the initial frame of the first animation sequence and the final frame of the eighth animation sequence are both the first

key frame.

Since these two frames, initial and final, of the walk cycle are identical, then just by copying the same walk cycle over and over again an animation consisting of an arbitrary number of walking strides can be produced. This animation consists of a character walking in place on the screen. Now a horizontal translation can be introduced that will move the character across the screen. This basic walk cycle can now be used as a basic building block in other animation.

First, because of the artwork restrictions required by this procedure, other characters can very easily be animated through very similar walk cycles. The raster manipulation produced by the computer can remain constant and only the artwork need be changed. Therefore, the walk cycle could be developed for a policeman and then by changing the artwork of a policeman for that of a farmer, a farmer is made to walk. An arbitrary number of different characters could be animated in identical walk cycles merely by preparing a different pieces of input artwork. The artwork would define the character and the computer program would animate it.

There are a number of key parameters that define the walk cycle and control its character. One of these is the number of frames between each key frame. Increasing this number slows the speed of the stride and conversely, decreasing this number creates a faster paced walk cycle. Another important parameter is the amount of leg rotation, that is, how far the legs are rotated during each stride. This parameter helps to determine the

length of the stride. Larger rotation angles produce longer strides. Assisting to determine the length of the stride is the parameter controlling the amount of leg bend. The amount of leg bend helps to determine just where the foot strikes the walking surface.

There are other parameters that can control the character of the walk. One of these is the amount of arm rotation. The amount of arm swing is a large factor in determining the character of a walking figure. Associated with this is the parameter of arm bend. Several other parameters are the amount of vertical body movement and vertical head movement. All of these parameters can be used to establish the personality of an animated figure. A certain combination can produce a cheerful bouncy personality. Another combination can depict a depressed cheerless personality. Other combinations can produce an aggressive, timid or another type of personality.

All of these different types of walk cycles that describe the various personalities can have their beginnings in the same basic walk cycle. A cheerful walk cycle would have its beginning in the nominal walk cycle. Then the parameter controlling the speed of the stride, the number of frames between key frames, can be adjusted to give a faster stride. Leg rotation would be increased slightly as would body and head bounce. Arm rotation would be increased even more. Arm and leg bend are not as important in this case.

Other types of walks that could be produced are a listless shuffle, a worried hurried walk, a strut and a sly inconspicuous walk. All of these episodes would have their



start in the basic walking stride. The differences could be stored in a memory or programmed at the moment of need.

#### 7.5 Implementation of a Hierarchical Walk Program

A hierarchical program can be written for the CAESAR system that inserts the proper or desired numbers into the correct locations. For a walk cycle that is composed of eight key frames, only the arguments for eight frames need be specified. The spacing between the key frames is also specified. CAESAR then calculates the parameters for the rest of the frames in the cycle in its usual manner. Another bit of information is the number of basic cycles that are to be copied so as to have a specified number of walk cycles.

For example, all of the angles that the legs assume in the eight key frames are related to a single parameter. When this parameter is specified then the program assigns the proper rotation argument to the legs in each key frame. The same procedure is then followed for the other parameters whose arguments can change the nature of the walk cycle. The initial value of each control parameter is predetermined and that argument is used if no other is specified. The number of frames between each of the key frames can be a constant or can vary. Whatever produces the best effect can be used.

The basic walk cycle program contains the parameters of the eight key frames with nominal values. There is also a controlling portion of the program. The angles for the leg positions are dependent on a control parameter that is specified when the walk cycle program is used. This control parameter is appro-

priately translated to the proper leg angular position for each key frame.

In a similar fashion, other control parameters determine the other key frame parameters such as arm angles, head and body vertical motion, leg bend and arm bend. The number of frames to be inserted between each key frame is also specified. Finally, the number of cycles of the walk to be produced are also specified. The walk program then uses the values assigned to the control parameters to "draw" the key frames. The specified number of frames are placed between the key frames and as in normal operation the computer calculates all of the parameters for each of the frames between the key frames. Then the completed walk cycle is copied the specified number of times.

A program can be written that produces a walk cycle by utilizing the basic animation equation.

$$\bar{W} = \sum_{\alpha=1}^{8n} \bar{T}[X_{\alpha}, Y_{\alpha}] \bar{M}[A_{\alpha}, B_{\alpha}] \sum_{\beta=1}^m [h(q-T_{\beta-1}) - h(q-T_{\beta})] \bar{T}[X_{\alpha\beta}, Y_{\alpha\beta}] \bar{R}[\theta_{\alpha\beta}] \bar{T}[J_{\alpha\beta}, K_{\alpha\beta}] \bar{M}[a_{\alpha\beta}, b_{\alpha\beta}] \bar{Z}_{\beta} \quad (7.1)$$

Certain assumptions are made:

1. All characters are composed of six separate sections so that  $m=6$ .
2. The key frames are evenly spaced throughout the walk cycle so that  $n$  is a constant parameter.
3. The artwork is restricted so that each particular member is always on the same section. Also the center of rotation of each member is located at a fixed position on the input art-

work. Each section size remains constant which means that

$T_1, T_2, T_3, T_4, T_5$  and  $T_6$  have fixed values.

The steps in using this hierarchical walk cycle program are as follows:

1. The artwork is properly positioned for input scanning.
2. Nominal values for the first key frame parameters are used by the computer to assemble the character. This means that pre-specified values for  $X_1, Y_1, A_1, X_{11}, Y_{11}, \theta_{11}, J_{11}, K_{11}, a_{11}, b_{11}, X_{12}, Y_{12} \dots X_{16}, Y_{16}, \theta_{16}, J_{16}, K_{16}, a_{16}$  and  $b_{16}$  are used.
3. The character is examined by the operator and certain parameters such as  $J_1, K_1, a_1$  and  $b_1$  can be re-adjusted to conform to the particular artwork. A tall character would need these parameter values to be different from those of a short character. These re-adjusted parameter values are then substituted for the preset values in the basic program.
4. The eight key frames of a typical walk cycle are exhibited in Figure 7.15. The maximum amount of leg rotation is chosen by the operator. This parameter value is the negative value for the left leg in the third key frame, the right leg in the seventh key frame and is stored directly in the appropriate memory locations. As a reference, a leg that is vertical has a zero rotation angle.

The rotation of the right leg in the seventh key frame is the negative maximum value. In the third key frame the right leg has 66% of the maximum leg rotation, the maximum positive rotation. To summarize, the leg rotations in each key frame with respect to the maximum are:

| Key Frame | Right Leg | Left Leg |
|-----------|-----------|----------|
| 1         | - 11%     | - 23%    |
| 2         | 34%       | - 69%    |
| 3         | 66%       | -100%    |
| 4         | 29%       | - 37%    |
| 5         | - 23%     | - 11%    |
| 6         | - 69%     | 34%      |
| 7         | -100%     | 66%      |
| 8         | - 37%     | 29%      |

Therefore the memory locations storing the value of the rotation parameter for the leg sections are loaded either with a value representing a function of a nominal value or of value chosen by the operator.

5. Arm rotation is determined in the same manner. A major difference is that the right arm has the same percentage of the maximum rotation as the left leg and vice versa. However, the amount of the maximum rotation can be different for the arms with respect to the legs.
6. The amount of body bounce can be specified. Let the maximum positive vertical displacement be 100% and the minimum be 0%. Then key frame numbers 1 and 5 have 100% vertical displacement, 2 and 6 have 70%, 3 and 7 have 0% and 4 and 8 have 20%. Body bounce and leg rotation are related since a greater leg rotation will require a greater body bounce to keep the feet on the floor.
7. Head bounce is very similar to body bounce. The full amount of vertical movement for the head may be less than, equal to

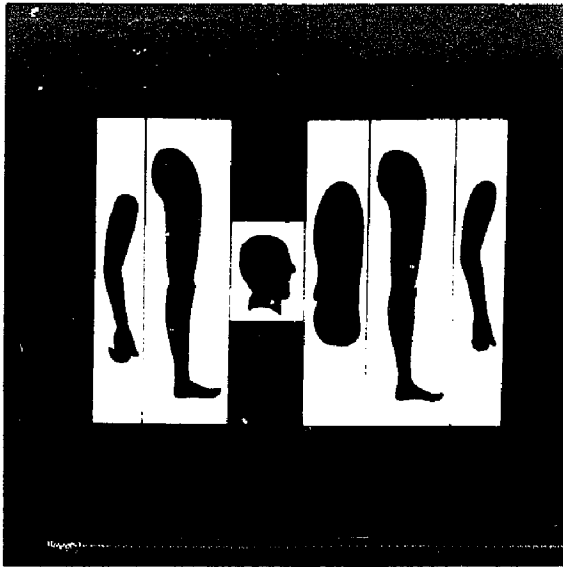
or more than the amount for the body. The desired amount must be specified.

8. The amount of bending in the leg sections will be related both to the amount of leg rotation and to the body bounce. The amount of bend appropriate to the character must be specified. There is a difference in the values of the desired amount of leg bend and leg rotation. Initially there is 0% bend in the right leg in frames 3 and 5. Maximum bend occurs in frame 8 with 80% in frame 1, 20% in frames 2 and 6 and 10% in frame 4. For the left leg the same percentages hold only offset by 4 key frames.
9. This step is very similar to the previous one as the desired amount of arm bending must be specified. Unlike the rotation parameter the arm and leg bend amounts are the same in each key frame.
10. The speed of the walk cycle is determined by the number of frames between each key frame. A smaller number of frames between key frames will produce a faster walk and conversely, more frames will make the walk cycle slower. The spacing between each of the key frames is initially identical and determined by the specified parameter. Special effects can be achieved by varying the spacing between each key frame. For example, a type of limp could be produced by using more frames between key frames three to four, four to five, five to six and six to seven than between the other key frames.
11. Finally, the number of walk cycles can be specified.

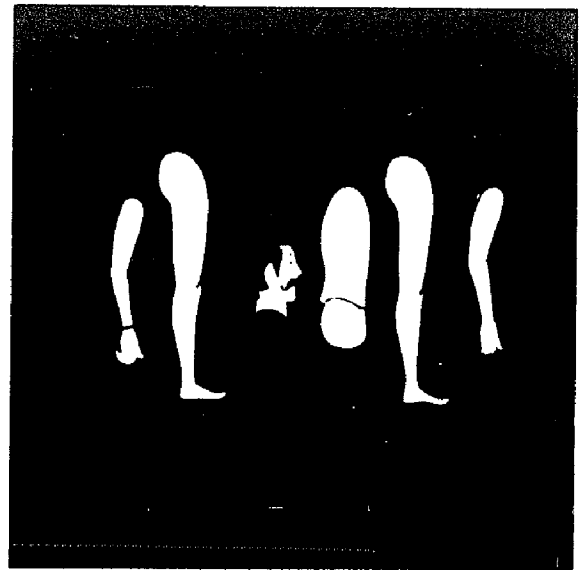
Thus, an arbitrarily long walking sequence can be programmed with

the animator needing to make very few decisions. However, he can change the nominal walk cycle by modifying any of the parameters for any or all of the key frames. The computer always handles the problem of drawing the "in between" frames and fairs the key frames into a smooth continuous movement.

Photographs of the actual key frames of a walk cycle animated by CAESAR are shown in Figures 7.16 through 7.25.

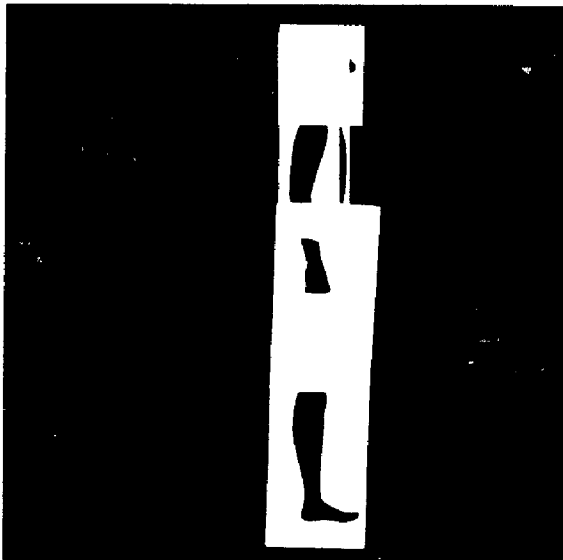


(a)

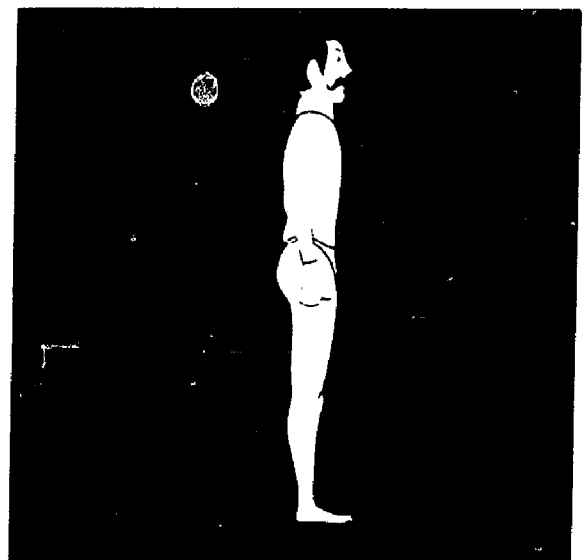


(b)

Figure 7.16 Input Artwork Configuration for Pierre  
 (a) Negative of Artwork on Raster  
 (b) Positive of Artwork

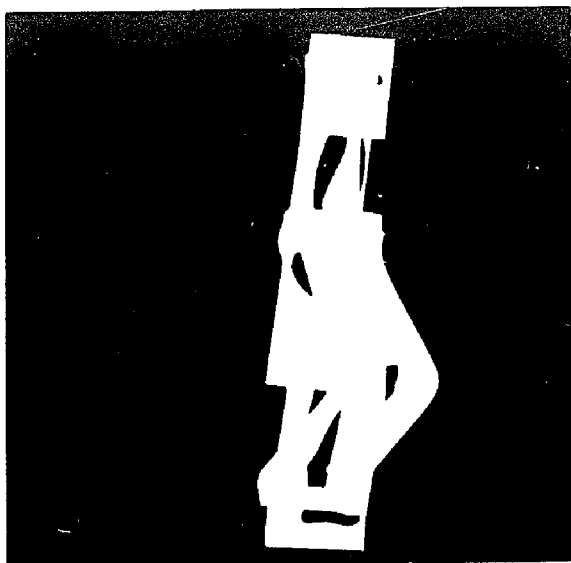


(a)

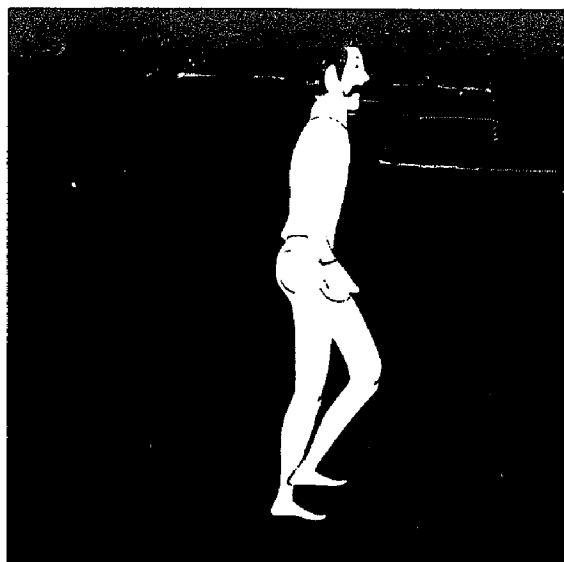


(b)

Figure 7.17 Pierre as Assembled  
 (a) Negative  
 (b) Positive



(a)

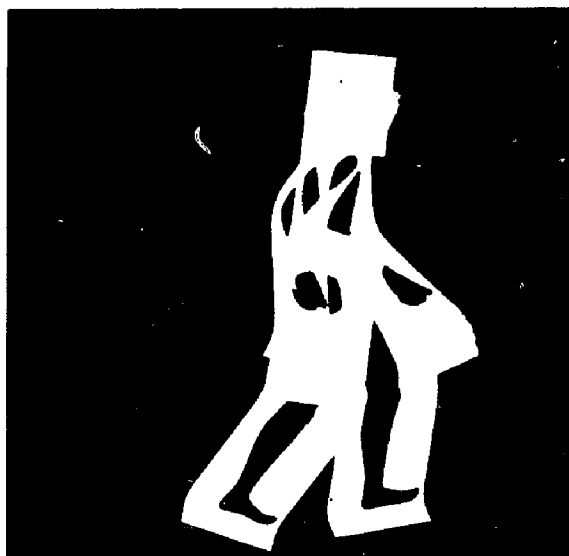


(b)

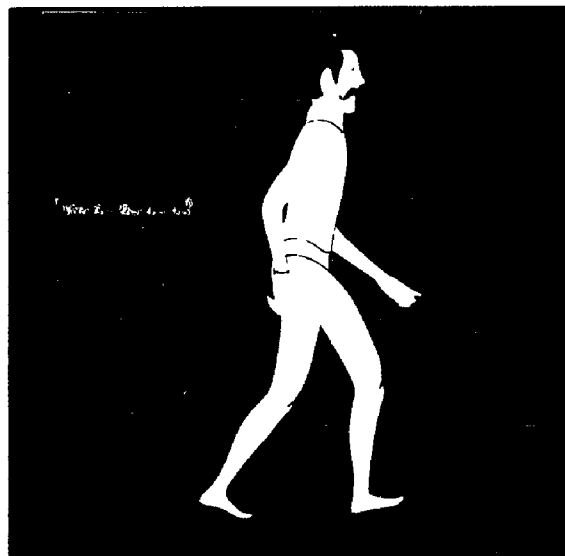
Figure 7.18 First Key Frame of a Walk Cycle

(a) Negative

(b) Positive



(a)



(b)

Figure 7.19 Second Key Frame of a Walk Cycle

(a) Negative

(b) Positive





(a)

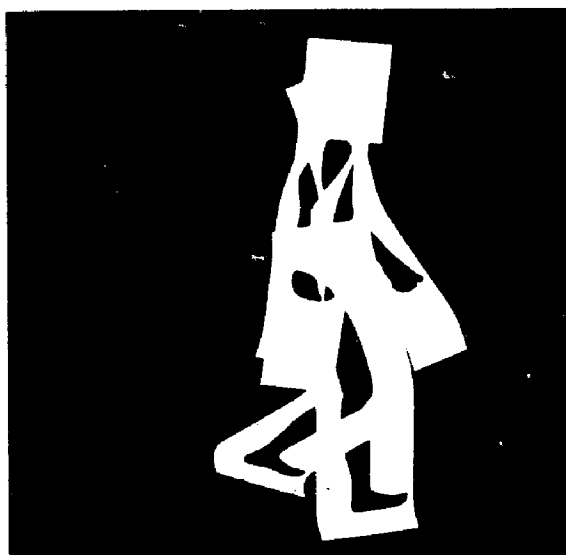


(b)

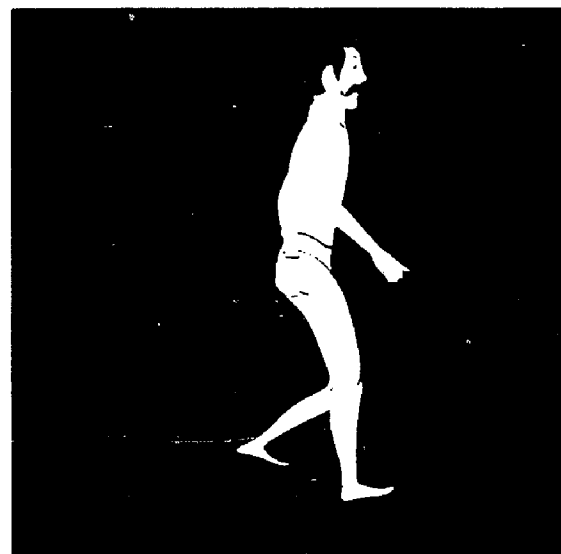
Figure 7.20 Third Key Frame of a Walk Cycle

(a) Negative

(b) Positive



(a)

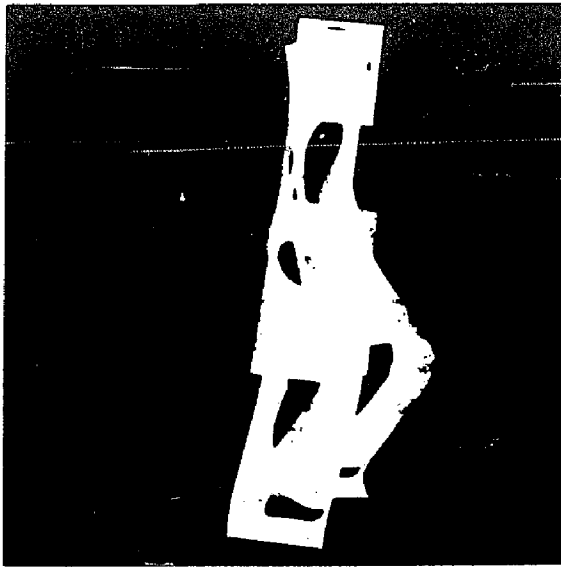


(b)

Figure 7.21 Fourth Key Frame of a Walk Cycle

(a) Negative

(b) Positive

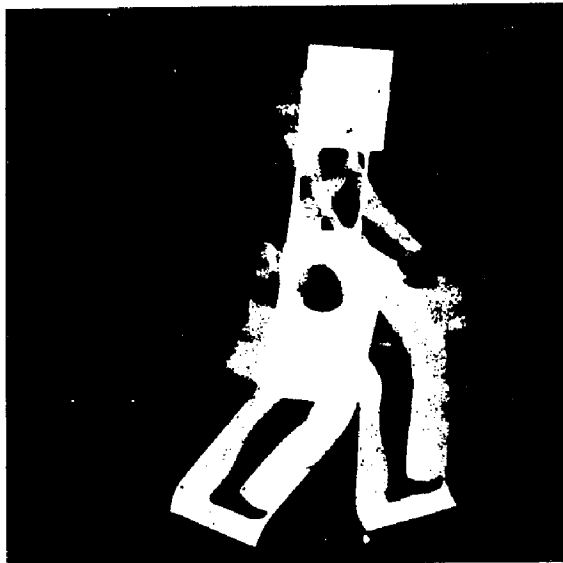


(a)



(b)

Figure 7.22 Fifth Key Frame of a Walk Cycle  
(a) Negative  
(b) Positive

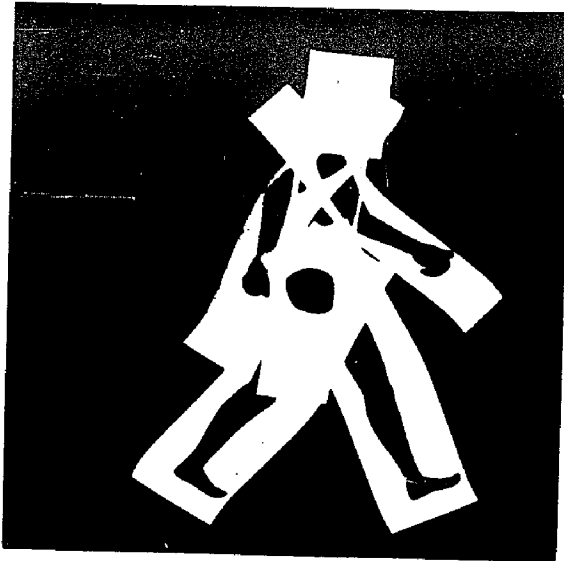


(a)



(b)

Figure 7.23 Sixth Key Frame of a Walk Cycle  
(a) Negative  
(b) Positive

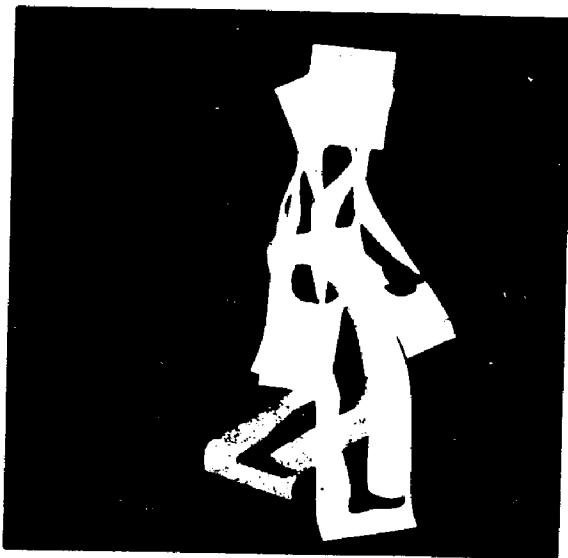


(a)

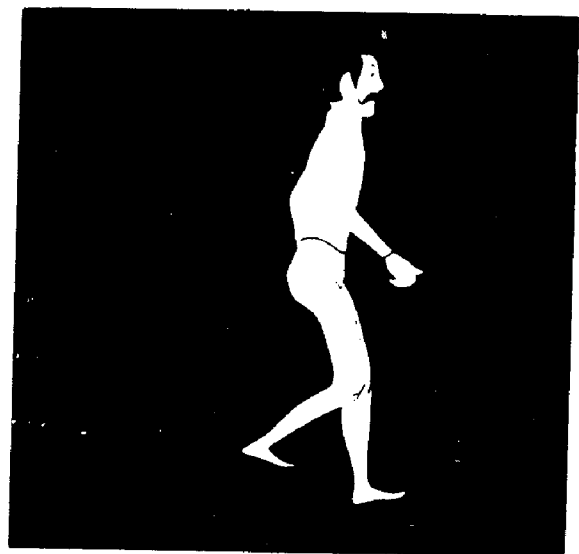


(b)

Figure 7.24 Seventh Key Frame of a Walk Cycle  
(a) Negative  
(b) Positive



(a)



(b)

Figure 7.25 Eighth Key Frame of a Walk Cycle  
(a) Negative  
(b) Positive

## 8. SUMMARY AND CONCLUSIONS

### 8.1 Summary

The increasing use of animation in the areas of entertainment, education and advertising has been discussed. Furthermore, cost has been pointed out as the factor limiting its growth. This cost has been steadily increasing due to the large amount of labor needed to produce animation. As a result various techniques have been employed to decrease this cost by reducing the amount of labor required per unit of animation.

Among the methods used to aid in the production of animation the most promising has been the use of computers. Systems that employed conventional digital computers were mentioned. Much effort has been expended by many talented people to draw pictures with machines designed to work with numbers. This effort has produced some very unique results but in general computers have not produced much commercial animation.

A new technology employing the technique of mapping planes was then described. It was shown how an experienced animator's skills could be utilized by this system. The quality and control of the output image could surpass that of any other computer aided animation. An animator could use his drawing skill to program the imagery and his skill in creating illusions to program the motion. Relative to other computer animation systems very little data was needed to describe the animation. This is due to the use of nonchanging image information combined with the small amount of parametric data needed to control the movement.

Next, it was shown how the mapping functions control-

ling the animation could be instrumented. Television techniques combined with a special purpose hybrid computer made the system realizable. Analog portions of the computer produced the animated rasters and the video portion transferred the imagery from the input artwork to the animated raster on the output display. The digital portion was used to control the analog section and also to store data.

Finally a method of programming complex character animation was discussed. The technique of hierarchical programming was shown to be very powerful. A basic walk cycle could be pre-programmed. Then an animator need only call up the program, specify certain key variables and he would have a walk cycle. It could then be modified to individualize it.

Other people had discussed many of the steps necessary to produce an animated film. Falk (1941), Halas (1959) and Deily (1968) did a good job of making one aware of the many facets to a successful production. As they explain, there is much more to an animated film than merely drawing pictures. The story itself, development of the characters, determination of the timing, creation of the storyboard and production of the sound track are all tasks that take much skill. A system such as CAESAR is intended to be a tool that can perform the drudgery of repeated picture drawing.

## 8.2 Areas for Research

Most of the capabilities that have been discussed have already been implemented in the CAESAR system. Much greater capabilities are possible that will make it even easier to produce

quality animation. First, the system is hybrid having both analog and digital computer components. It is possible to use only a standard mini-computer and still have a very powerful tool. Since the digital portion needs to calculate or otherwise produce only the section parameters, fast computational speeds or capacity are unnecessary. However, a large, easily accessible memory does make it possible to have more routines pre-stored and this can cut seconds from the response time. So it is handy but not necessary.

Another improvement would be a digital memory that could store the input artwork and read it out at television video rates. The memory could be loaded by a slow scan television camera whose output video was quantized. This camera would scan the artwork and transfer the information into the digital memory. During the animation sequence the memory would repeatedly be read nondestructively and would provide the image on the animated rasters. The same memory could store the essential parameters necessary to make a complete character from the individual segments. These parameters would be those of raster size; center of rotation; its magnification factor; the type of shaping signal and its frequency, amplitude and phase; its angular orientation and its position in the frame. Thus, one could call up a particular character and the system would create it on screen fully assembled and ready for animation. If certain characters are to be re-used many times, like in a television series, then a library of their various poses could be assembled. As the different poses are needed for the storyboard they could be recalled and incorporated into the production.

A subject that should be studied in the future is distortion limits. The mapping technique produces animation by manipulating the imagery contained on the individual sections. Some of these manipulations are due to nonconformal mapping functions and distort the image. As long as these distortions are small, viewers continue to perceive the desired image. However, at some point, as the amount of distortion increases, the image loses credibility. As was previously discussed, an arm bent only by the translation operator becomes distorted beyond recognition when bent more than about 100 degrees. The point at which the distorted image of an arm fails to convey that image has not been determined. It seems that there might be a relationship between the extent the operator departs from being conformal and the point where the image loses credibility.

Perhaps it is unnecessary to be concerned about how big a plane must be before perspective is necessary to maintain the illusion of flipping. It may be possible to create the illusion of a head turn with just two or three different views and appropriate cuts to the views. More in between drawings may be redundant. Answers to these and other related questions could both simplify computer animation and increase the consistency of its quality.

There is also much promise in the area of digital signal processing. Because of the small amount of data needed to specify each output image it is possible to do some real time data processing with a relatively simple digital computer. As an example, tests have shown that a software center of rotation can

be selected that is independent of the hardware center of rotation. This makes it possible to assemble a character from a number of sections, each with its own center of rotation. Then another center of rotation can be assigned to the entire character and all of the assembled sections can be rotated together.

Another example would use the digital computer to tie various sections together. A face could be composed of several sections containing the artwork for eyes, a mouth and a nose. These sections could be identified as belonging to the "face" and then whenever a "face" parameter was changed all of the individual sections of the "face" would move or change together.

### 8.3 Conclusion

Animation if it is to serve any major function in the future will involve a computer. The similarities between successive frames are very much like the solutions of an iterative process. Among the many techniques being employed to produce animation with a computer, I feel that the most promising is the mapping process just discussed.

The animation systems that have used this technique have been quite successful in producing animation ranging from conventional character animation to unique abstract graphic animation. This technique can create animation in real time giving the animator immediate visual feedback. This visual feedback is an important property that a number of pioneers in computer animation have recognized (Honey, 1971), (Baecher, 1969), (DeFanti, 1973). The combination of image programming by the artist, well organized control and immediate visual feedback make the CAESAR system the



most advanced tool in the field of animation. Furthermore while the concept of animation by mapping is realized quite well by the CAESAR system it has the potential for expansion to provide an even more powerful tool.

An animation system employing a hybrid computer and utilizing some of the concepts discussed in the main body of the dissertation has been constructed by the Computer Image Corporation. It has been named CAESAR, an acronym for Computer Animated Episodes by Single Axis Rotation. The name of the system emphasizes its prime characteristics. The animation is produced and controlled by a hybrid computer as a series of episodes. Rotation is limited to that about the Z axis.

The system basically includes an analog portion for generating output signals representing from one to eight sections of a raster on which images viewed by a video camera can be produced. Analog inputs to the analog portion define the structure, size, shape, location, orientation and other parameters of the raster section to effectively define the shape of each part of the viewed image produced thereon. By varying the analog inputs, the raster section parameters can be made to vary, thus imparting motion to the image. In this system the values and variations in values of the analog inputs are controlled by digital signals from a digital computer which establishes these digital control signals from information fed to it from a director.

More specifically, the analog computer portion generates X and Y coordinate signals representing each section of an animated image for each frame of an animated sequence, each section of the image comprising a raster section. Sweep generators within the analog portion generate the basic horizontal and vertical sweep signals which are combined with, or modulated by

other input signals to define the structure, shape, size, position and other parameters of each raster section for each sequence frame. The X and Y coordinate signals at the output of the analog portion are applied to a cathode ray tube to produce a display of the animated sequence.

The input signals to the analog portion define, for each raster section, parameters such as X and Y position, X and Y size, horizontal and vertical axes of rotation which determine the center of rotation of each raster section, section depth, cosine and sine of the angle of rotation, intensity, and horizontal, vertical and depth modulation. Other parameter inputs define overall X and Y position and depth for the entire image. All of these parameter input signals act in conjunction with the sweep generators to ultimately produce the X and Y coordinate output signals from the analog portion for use in producing an animation sequence. By varying these input signals, each raster section can be made to vary in height and width, move anywhere in relationship to any other raster sections, rotate about any point located inside or outside the raster section, and can be modulated with the variety of modulation signals to produce bending or distortion of the raster section. The bending or distortion can be made to vary by varying modulation signal parameters such as frequency, type of waveform, phase and amplitude.

The parameter signals of the analog portion and the modulation signal parameters are established for each section of the image and for each frame of the sequence by a digital computer portion. It automatically calculates these parameters from infor-

mation it receives at its input from a director. Parameter data for each raster section for initial and final frames of a sequence of a selected number of frames are selected on the director and this information is fed to the digital computer. The digital computer is programmed to automatically calculate, upon command, the parameter data for each section and for each frame between initial and final frames in accordance with a selected fairing function. It must also store this information in a digital memory such as magnetic tape or disc. Depending on the fairing function selected, these digital computations may be linear or based on some other mathematical function which will define the patterns of parameter change throughout the sequence. For example, in this way an arm of a figure can be made to move at a constant rate depending on the fairing function selected. The digital information recorded on the magnetic tape or disc can then be played back through the digital computer to the analog portion to produce the animated sequence.

When a figure is displayed on the animated raster sections a video camera is trained to scan the individual parts of the figure to be animated. Timing pulses are generated to time the scan of the video camera in synchronization with the production of the X and Y coordinate output signals from the analog portion to reproduce each part of the scanned figure on a raster section as defined by the X and Y coordinate output signals. The result is that the video signals from the video camera determine the detail surface characteristics and shape of each part of the image as displayed on a raster section. The analog portion of the computer defines the structure, shape, size, position and

other characteristics of each raster section as determined by the values of its parameter inputs. Digital signals from the digital computer vary the parameter inputs in a controlled manner between initial and final frames to animate the raster sections and hence, the parts of the figure produced on them to create the animation sequence.

With this system the editing can actually be done by the digital computer and not after the scenes are recorded on video tape or film. By dividing a sequence to be filmed into segments or scenes having initial and final frames, the information for creating each frame of the entire sequence can be recorded in digital magnetic tape as the scenes are produced. The information can then be played back through the system for ultimate display and recording of the animation sequence. In this way, a sequence of any length can be produced without the need for post editing.

The CAESAR system has some other very powerful capabilities that make it a useful animation tool. First, a method of converting the various gray scales of the artwork into any arbitrary color as viewed on a color television monitor is used. This makes it possible to have a character wear a red shirt, blue pants and black shoes in one scene and then with the same artwork appear to have a brown shirt, green pants and purple shoes in another scene.

A system called overlap solves the "hidden line" or "hidden surface" problem. This problem exists whenever a two-dimensional representation of a three-dimensional object is produced on a CRT. Overlap is a unique hardware solution using analog

and video technology. It makes it possible to draw figures consisting of opaque surfaces in real time.

Extensive use of video technology is made. A video key process is used to insert the character or object being animated into any arbitrary background that has been converted into a television signal. Ordinarily it is a piece of colored artwork viewed by a color television camera. The output of the system can be viewed on color television monitors and recorded on a video tape recorder. If film is desired then the video tape can be converted to film by any of several service firms.

From the foregoing, it is apparent that this system allows an artist to produce animated sequences in far less time than would be possible using conventional techniques (requiring twenty-four separate drawings for each second of animation) and yet provides him with wide control for producing many different types of animation. The result is that the artist animates with the computer by operating the director controls, giving his full attention to creativity and results rather than the tedium of producing many similar pieces of art.

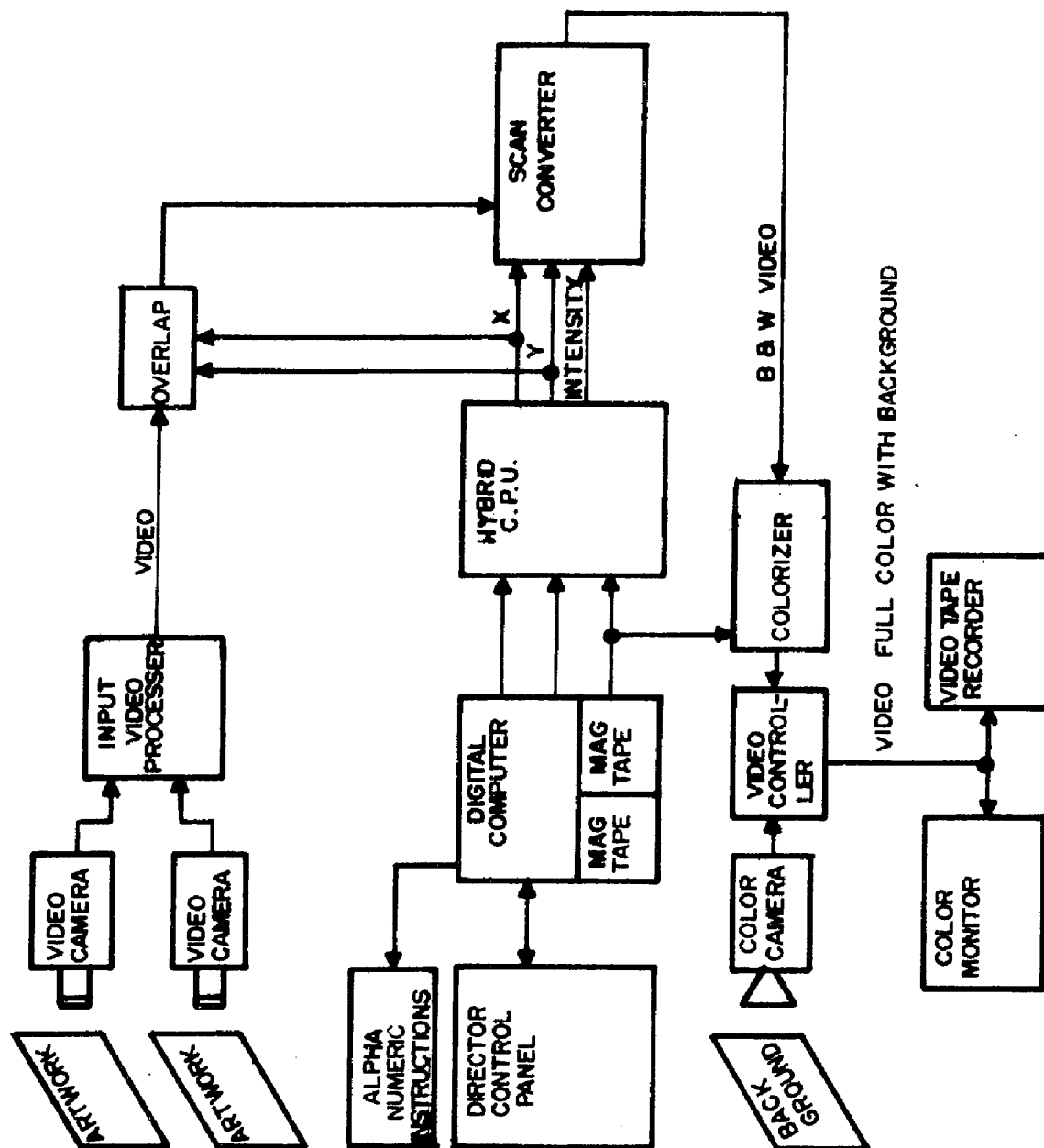


Figure 9.1 CAESAR Animation System Block Diagram

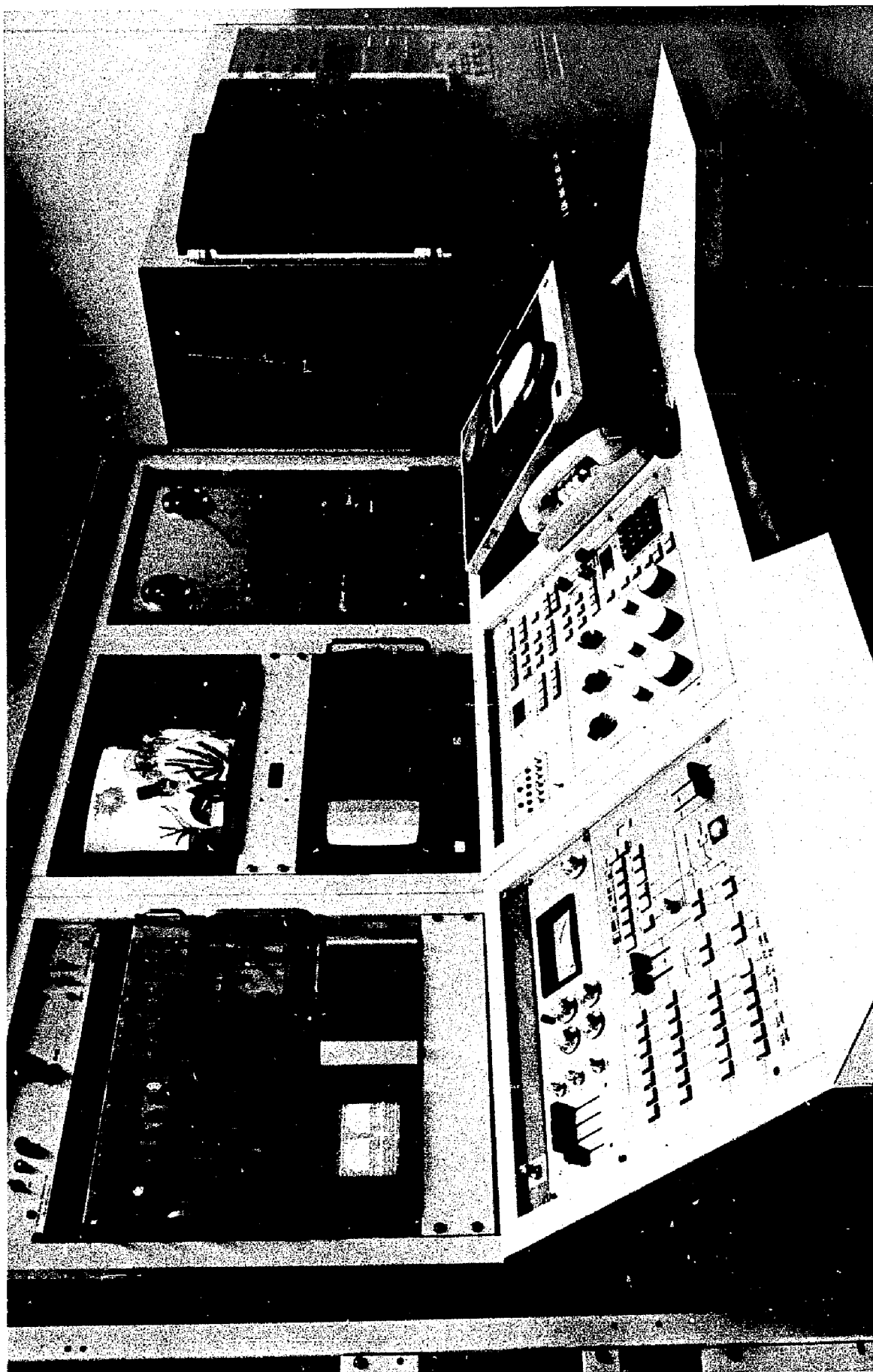


Figure 9.2 Control Console for CAESAR



10.        BIBLIOGRAPHY

1. Baecker, Ronald M. Picture-driven animation. Spring Joint Computer Conference Proceedings, 1969, 273-288
2. Bereniji, I. and H. Busley. Animation by computer. Computer Aided Design, Winter, 1971.
3. Blair, P. Animation. Tustin, California: Foster Art Service, Inc., 1949, 24-34.
4. Churchill, R. V. Complex Variables and Applications. New York: McGraw-Hill, 1960, 174-181.
5. Davis, Douglas. The Artist and the Computer. Newsweek, September 13, 1971, 78-81.
6. DeFanti, T. The Graphic Symbiosis System. Real-Time Film Animation. Annual Report to the National Science Foundation Office of Computing Activities, Grant Number GJ-204, January, 1973.
7. Deily, D. Principles for Producing Computer Animated Motion Pictures. Doctoral Dissertation at the Moore School of Electrical Engineering, University of Pennsylvania, Philadelphia, Pennsylvania, 1968.
8. DeJoux. Major Breakthrough in Animation Process. Boxoffice, April 12, 1971, 8-10.
9. Falk, Nat. How to Make Animated Cartoons. New York: Foundation Books, 1941, 9.
10. Franken, Jerry. The Walter Lantz Story. American Cinemeditor, Fall, 1971, 26-28.
11. General Electric Company. Film - Hancock Airport. Share XL Computer Graphics Film Festival, March 1973.
12. Groves, B. R. The Use of Computer-Generated Motion Pictures in the Analysis of Electrical Engineering Phenomena. Doctoral Dissertation at The Ohio State University, Columbus, Ohio, 1966.
13. Halas, John and Roger Manvell. The Technique of Film Animation. New York: Focal Press, 1959, 27, 319.
14. Halas, John and Walter Herdey. Film and TV Graphics. Zurich: The Graphis Press, 1967, 90-91.

15. Halas, John. Art in Movement: New Directions in Animation.  
New York: Hastings House, 1970, 96.
16. Harrison, L., F. J. Honey, E. J. Tajchman, M. M. Parker.  
Digitally Controlled Computer Animation Generating System.  
United States Patent No. 3,747,087, July 1973.
17. Honey, Francis J. Artist-Oriented Computer Animation.  
Journal of the SMPTE, March 1971.
18. IBM, The art of motion graphics. Computing Report in  
Science and Engineering, March 1969, 10-13.
19. Kallis, Stephen A., Jr. A Computer-Controlled Animation  
Stand. American Cinematographer, September, 1969, 870-906.
20. Kaplan, Wilfred. Operational Methods for Linear Systems.  
Reading, Mass., Addison-Wesley Publishing Company, Inc.,  
1962, 46-47.
21. Knowlton, Kenneth C. Computer Films. Filmmakers Newsletter,  
December, 1970, 14-20.
22. Lehman, Patricia R. Design Exploration into Computer Graphics  
for Animated Films and Video Tapes. Unpublished thesis  
at University of Kansas, 1972, 17.
23. Madsen, R. P. Animated Film. New York: Interland Publishing,  
Inc., 1969, XI, 3, 4, 9, 39.
24. Madsen, R. P. The Impact of Film. New York: Macmillan  
Publishing Co., Inc., 1973, 147-171, 455, 487-489.
25. Miura, T., J. Iwata, and J. Tsuda. An Application of Hybrid  
Curve Generation-Cartoon Animation by Electronic Computers.  
Spring Joint Computer Conference Proceedings, 1967,  
141-148.
26. Murray, M. P., A. B. Drought and R. C. Kory. Walking  
Patterns of Normal Men. The Journal of Bone and Joint  
Surgery, Vol. 46-A, No. 2, March 1964, 335-360.
27. Sutherland, Ivan E. Computer Displays. Scientific American,  
June, 1970, 56-81.
28. Talbot, P. A., J. W. Carr, R. R. Coulter and R. C. Huang.  
Animator: An On-Line Two-Dimensional Film Animation  
System. Communications of the ACM, April, 1971,  
251-259.
29. Warnock, J. E. A Hidden Surface Algorithm for Computer  
Generated Halftone Pictures. Doctoral Dissertation at  
University of Utah, Salt Lake City, Utah, 1969.

30. Wurman, Richard Saul. Fetter's Computer Graphics. Design Quarterly 80, Walker Art Center and the MIT Press, 1971, 29.
31. Wylie, C. R. Advanced Engineering Mathematics. New York: McGraw-Hill, 1951, 383-395.
32. Zajac, E. E. Computer Animation: A New Scientific and Educational Tool. Journal of the SMPTE, November 1965, 1006-1008.

COMPUTER AIDED ANIMATION UTILIZING  
A MAPPING PROCESS

---

An Abstract of a Dissertation  
Presented to  
The Faculty of the College of Engineering  
University of Denver

---

In Partial Fulfillment  
of the Requirement for the Degree  
Doctor of Philosophy

---

by  
Edwin J. Tajchman  
April 1974

## ABSTRACT

Since its discovery, animation has become increasingly effective in the areas of advertising, entertainment and education. However, the steadily increasing costs of conventionally produced animation have limited its use. Many different techniques have been developed that can reduce the work necessary to produce animation and thereby lower its cost. The most promising of these techniques has been the application-of computers to the animation process.

A new technology using a hybrid computer to produce animation by a mapping process has been developed. The output image is produced by mapping the input artwork onto a complex surface and is controlled by controlling the parameters of the mapping operators. Only three operators are required to describe the mapping. Translation, magnification and rotation operators with appropriate arguments are sufficient to shape the output image. Time varying arguments produce images changing with time, or animation. Mapping operators whose arguments are functions of position on the input plane extend the shaping capability of this technique.

A notation for a general mapping operator is defined and the necessary three types of operators developed within that format. A combination of the three operators make it possible to map artwork of a character onto a changing output plane and thereby produce an animated character. It is then shown how the different operators can be implemented with a hybrid computer. A combination of television, digital and analog subsystems make the mapping technique very realizable.

Additional capability can be achieved by using compound and hierarchical programming. Complex image manipulations are much easier to control if certain compound operators are instrumented. But even more powerful is hierarchical programming. Many animation sequences are repeated many times such as walk cycles. It is possible to pre-program a routine so that any character conforming to certain specifications can be easily animated in a walk cycle. The nominal walk cycle can then be modified to suit the character's personality by changing a few parameters.

Examples in the form of key frames from an animated walk cycle are shown. Areas needing more investigation are discussed. A hybrid computer animation system named CAESAR is described.